CHARACTERIZATION OF ANAEROBIC BIOREACTORS FOR BIOENERGY GENERATION USING A NOVEL TOMOGRAPHY TECHNIQUE

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The animal waste (biomass) created by the growth of the livestock industry poses both: environmental problems and opportunities. Viewed more positively, the waste can be a valuable source of affordable, sustainable, and renewable bioenergy in the form of methane (from biogas) generated from degradation. Anaerobic degradation of animal wastes is a known waste treatment technology to produce methane for use as fuel. Amongst various mixing configurations of anaerobic bioreactors for degradation of animal waste, bio gas-lift bioreactors are favorable for large scale operations and offer many advantages. Phase holdup distribution in the flow field affects the mixing characteristics and by extension the performance of gas – lift anaerobic bioreactors.

Anaerobic bioreactors like most multiphase systems are opaque. Tomography is a powerful technique capable of determining the effect of operating and design parameters on the phase holdup distribution by visualization in opaque flow systems in a noninvasive manner. In this work, a novel dual energy – dual source computed tomography (DE –
A DSCT) system capable of imaging phase holdup distribution in three phases flow was successfully developed and validated. A new image reconstruction methodology was applied for the validation of the DE – DSCT which enables accurate imaging of the phase holdup distribution in large scale multiphase systems. Radioactive particle tracking and tomography have been applied to characterize the effect of operating parameters and sparger design on the liquid flow behavior of laboratory scale gas-lift anaerobic bioreactors. Based on the results, the bioreactor was scaled up to a pilot scale. Performance studies were carried out in the pilot scale bioreactor to optimize mixing power input and biogas production.

The DE-DSCT technique developed as a part of this dissertation provides an effective research tool for expanding experimental research in a wide range of multiphase flow systems via imaging. It has overcome many limitations common to the various tomography techniques that exist for imaging systems with three phase flow. The knowledge gained from anaerobic bioreactor studies improves the fundamental understanding of the effects of design and hydrodynamics of anaerobic bioreactors for optimized bioenergy generation in the form of methane.
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Chapter 1

Introduction and Motivation

1.1 Introduction

The ever increasing energy requirements to support a modern lifestyle have lead to unprecedented pressure on the environment. Green house gasses emitted by the consumption of fossil fuels from sources such as coal and crude oil are responsible for rise in global temperature. The rise in temperature could have devastating repercussion on the environment. The energy consumption is expected to double in the coming few decades. The fast growing economies of China and India have created a demand for higher living standards comparable to the western world. This in turn has triggered a tremendous demand for energy further increasing the need for energy. Fossil fuels reserves – the main staple of energy supply – are expected to deplete with the projected demands. Hence, finding a clean and abundant source of energy that could satiate the energy demands of the modern life style with a minimal environmental foot print is one of the biggest challenges of this century. It has been well established that renewable sources of energy have a minimal environmental foot print as compared to other nonrenewable sources. It is imperative that research efforts should be directed towards harnessing renewable sources such that they become the staple source of energy.
Close to 100 quadrillion BTU of energy was consumed by USA in 2006 (EIA 2007). About 32% of this consumed quantity was imported (Figure 1-1). Fossil fuels which is a nonrenewable source, accounted for 85% of the consumption. This is a clear indicator of the large dependence on nonrenewable sources of energy that are known to be depleting and have a significant environmental footprint. Only 7% of the total energy consumed was obtained from renewable sources. While this indicates a huge gap in energy production from renewable sources and energy consumption, it also highlights the urgent need to expand research and development efforts in the area of renewable energy.

Figure 1-1: Energy flow statistics for year 2006 (Source: EIA, Annual Energy Review 2006)
The production from renewable sources has increased two fold over the last fifty years (Figure 1-2B) and is projected to grow faster in the coming decades. A major source of renewable energy source is hydroelectric power and wood (Figure 1-2A). A smaller portion of the source is from waste and biofuels. There has been tremendous growth in the area of biofuels recently with advancements in the area of generating ethanol from cellulose and corn. Biomass, such as agricultural waste and plant residue that contain high quantity of cellulose, is eventually converted to ethanol though enzymatic hydrolysis and fermentation. Ethanol from biomass is a great renewable and clean substitute to petroleum and natural gas based liquid fuels for transportation purposes. Waste is the other source of biological renewable energy. Methane generated from the anaerobic digestion of municipal and farm
based animal waste and methane recovered from landfills, could be used for heating and electricity generation. It also could be converted to synthesis gas and used as a feedstock for Fischer-Tropsch’s process. Generating energy from waste provides the added benefit of reduction in the environment pollution or the environmental footprint of the process that generate the waste. Although currently energy from waste contributes a small percentage towards the total national energy demand, it has a tremendous potential for further research and expansion. The positive spin off is that it generates energy while disposing waste in an environmentally safe manner.

Animal and farm wastes from agricultural sources are sources of largely unexploited renewable energy. It is estimated that about 230 million tons of animal wastes (dry weight basis) are generated in the United States annually (Sheffield 2002). Most of it is discharged untreated, which causes many pollution related problems such as surface and groundwater contamination, foul odor, greenhouse gas, and problems caused by ammonia leaching. Methane, a gas with higher greenhouse potential than carbon dioxide, is generated from these wastes as part of the degradation process remains unrecovered. This methane could be recovered and consumed as an energy source and thereby reducing its potential greenhouse. The residues of such treatment are liquid and semisolid byproducts which can be used as soil conditioners.

The natural degradation process could be facilitated by anaerobic digestion in anaerobic bioreactors. Over the last couple of decades the anaerobic digestion process has been applied to different kinds of agricultural, animal, and industrial wastes (Gosh 1997; Speece 1996). The prime motivation for earlier farm installations was odor elimination (Lusk 1998) later energy crisis of the 1970 as a background the focus moved towards recovery of methane for use as an energy source. The major challenges with the performance of field
anaerobic bioreactors are issues related to mixing and design configuration (Lusk 1998). Mixing important functional benefits in anaerobic digesters, such as aiding the process of venting the biogas generated, preventing stratification, helping in achieving temperature and pH uniformity, and maintaining intimate contact between the bacteria and the substrate. In short, adequate mixing provides a uniform environment conducive to biological degradation of the waste. However, neither quantification of the digester’s flow behavior, nor the effects of mixing on the performance of anaerobic digesters are well understood or reported in the literature. Results from research on the effects of mixing on anaerobic systems are contradictory (Angenent and Sung 2001; Angenent et al. 2002; Hashimoto 1982; Ho and Tan 1985; Hoffmann 2005; Karim et al. 2005a; Karim et al. 2005b; Rivard et al. 1995; Smith et al. 1996). The results do not elucidate or quantify its effects on performance in a structured manner.

Like most multiphase systems, anaerobic bioreactors are also highly opaque due the nature of the wastes treated, the dispersed gas, and the walls of the digester. Experimental techniques that are noninvasive are effective research tools to understand the effects of operational parameters on the hydrodynamics of such systems. Advanced noninvasive measurement techniques like γ ray computed tomography (CT) and computer automated radioactive particle tracking (CARPT) have been applied to help visualization in opaque multiphase systems. CT helps quantification of the time averaged holdup distribution of the phases via imaging, and CARPT helps quantify the flow field in terms of liquid circulation, turbulent parameters, and global and local residence times. Both techniques can be used concurrently to understand the hydrodynamics of the anaerobic bioreactor or any multiphase system.
Tomography techniques have been used successfully as a research tool to study effect of various operational parameters on the phase holdup distribution in the flow with in the system (Chaouki et al. 1997). It has evolved a power research tool as it is not hindered by the opacity of the flow, it is noninvasive, and has the ability to produce sectional phase holdup images of the flow. In effect it is able to provide an eye in an opaque flow. Of the various modalities that have evolved γ ray tomography has widest range applicability to different multiphase systems. The γ ray tomography technique has been used to study a gamut of two phase flow system that include gas – liquid, liquid – solid, and gas – solids flow. It has also been applied to gas – liquid – solid flow where the solid phase is stationary. However, anaerobic digesters, as well as a significant number of other chemical and biochemical systems, consist of three dynamic phases in the flow (gas – liquid – solid). The γ ray tomography technique used for two phase system can’t be used for three phase system due to an equation closure issue.

Unfortunately, research efforts directed towards the development of tomography techniques to image dynamic three phase systems have met with limited successes. The efficacy of the tomography techniques currently available for three phase flow doesn’t measure up to those used for two phase flow. The chief limitations have been due the type of photon sources used or the image reconstruction methodology followed to determine the phase holdup distribution. The dual energy γ and x ray tomography methods use low energy photons sources, thereby they can’t be used for larger domains (larger than 3 inches). The image reconstruction algorithms may not account for the stochastic nature of the γ or x photons thereby introducing error in the images. Dual modality methods that use γ ray tomography in conjunction with electrical tomography methods require investment in two fundamentally different types of experimental hardware. The electrical methods have limited
spatial resolution and rely on an undetermined system of equations. This brings in the uncertainties in the images obtained by the dual modality methods. Hence, the techniques for determining phase holdup distribution in dynamic three phase systems far behind its well established counterpart for two phase systems.

1.2 Motivation

There is a strong motivation for developing tomography technique that is capable of imaging three phase flow such that it could be applied to a wide variety of multiphase systems. The development process would include the development of the hardware for γ ray transmission data measurements, and the development of an effective image reconstruction methodology. Multiphase systems are ubiquitous in the process industry and are used for chemical and biochemical transformations (Dudukovic 2007). They are extensively used in various industrial sectors. Some of its energy related applications include, synthesis to natural gas conversion, biomass conversion to fuel and value added products and environmental besides anaerobic digestion of waste. A desirable tomography technique should: be capable of imaging large diameter system, not be limited by the by high attenuating material that constitutes either the wall of the system or the flow in the system, be capable of providing phase holdup images with high spatial resolution, prove accurate and reproducible results. Such a technique will benefit research efforts related in understanding the effects of operating and design parameters on the phase distribution in important multiphase flow systems in general. More specifically, in this study it would help determine the effects of design and operating parameters on the phase holdup distribution in anaerobic bioreactors. A dual energy dual source γ ray computed tomography (DE-DSCT) system using two
separate $\gamma$ sources emitting $\gamma$ photons of different energies capable of achieving these objectives would be developed.

To understand the effects of operating parameters on the liquid circulation and flow pattern in anaerobic bioreactors the CARPT technique would be applied. The information obtained from lab scale could be scaled to a pilot scale anaerobic bioreactor. Since these systems are expected to produce methane, system should be such that it generates methane in excess of the process energy it consumes. The multiple-radioactive particles tracking (MP-CARPT) technique developed by Vesvikar (Vesvikar 2006) complements the development of a DE-DSCT by determining the liquid circulation pattern and velocity data. The collective information and findings can be used to develop, design, and scale up anaerobic bioreactors and a variety of multiphase systems that could provide optimal design, scale up and performance. The unique data obtained by these techniques can be used to benchmark and evaluate computational fluid dynamics (CFD) based models and develop closures to further facilitate the proper design, scale up, and operation of such systems. Mechanistic of phenomenological models could be developed or validated based on data from DE-DSCT and MP-CARPT techniques.

1.3 Objectives

The overall objectives of this study are two prong. The first: to develop and validate a dual energy – dual source tomography technique that is capable of accurately imaging phase holdup distribution in three phase multiphase systems. This includes work related to the development of the hardware associated with technique, its automation, and the image reconstruction algorithms to obtain accurate phase holdup data. The second: to apply
tomography to study phase holdup distribution in anaerobic bioreactors, and apply computer automated radioactive particle tracking to determine the effect of phase holdup on liquid flow pattern. The knowledge gained will be used to conduct the performance studies on a scaled up anaerobic. Figure 1-3 shows the detail tasks associated with each aspect of the study and its integration to achieve the defined objectives of the dissertation.

![Diagram showing integration of objectives and tasks]

**Figure 1-3: Integration of objectives and tasks considered in this study**

The specific details of the development of tomography technique and other tasks related to this study have been described in the following sections.
1.3.1 Development of Dual Energy – Dual Source Computer Tomography

The objectives pertaining to the development of a dual energy – dual source computer tomography system to imaging three phase flow have been summarized as follows:

- Evaluation of the algorithms for image reconstruction and methodologies used for determining phase holdup distribution post image reconstruction, for two and three phase systems.

- Development of an advance algorithm for determining phase holdup distribution in three phase systems.

- Design and fabrication of a vertically oriented the dual energy – dual source computer tomography (DE-DSCT) automated experimental setup capable of imaging large scale multiphase systems.

- Validation of DE-DSCT setup and image reconstruction procedure with a two phase and a three phase phantom.

- Application of the DE-DSCT technique for determining phase holdup distribution in a selected dynamic three phase flow system including anaerobic digester and slurry bubble column.
1.3.2 Investigation of the effects of gas holdup distribution on the hydrodynamics of a biogas recirculated anaerobic digester

- CT investigation of the effect of the sparger design on gas holdup distribution in lab scale gas lift anaerobic digesters and characterization of the gas holdup distribution
- Study the effect of gas holdup distribution on the slurry recirculation velocity and mixing in a lab scale anaerobic bioreactor

1.3.3 Flow behavior and performance studies in a pilot scale anaerobic bioreactor

- Investigate the effect of operational parameters on gas holdup distribution in an 18 inch pilot scale anaerobic bioreactor using DE – DSCT technique
- Performance study on pilot scale bioreactors to investigate the impact of gas holdup distribution on the production of biogas (methane). Optimization of biogas production to mixing power input.

1.4 Thesis Organization

A review of the state of the tomography methods used to image phase holdup distribution in three phase systems has been provided in Chapter 2. A brief overview of biogas recirculated digester has been provided. Since Vesvikar (Vesvikar 2006) has reviewed gas lift reactors for anaerobic process exhaustively duplication has been avoided here.

Chapters 3 and 4 discuss the image reconstruction algorithms for two phase systems and limitations of algorithms for three phase systems. Improvements to image phase holdup in
three phase systems have been discussed in Chapter 4. Chapter 5 focuses on the development of the hardware and the electronics of the DE-DSCT systems. The validation of the DE-DSCT experimental setup and the image reconstruction algorithms with a three phase phantom are discussed in Chapter 5. Effect of sparger on the gas holdup distribution and its effect on hydrodynamics of a lab scale surrogate anaerobic bioreactor have been visited in Chapter 6. Performance studies carried out to optimize the performance of pilot scale anaerobic digester and the impact on the gas holdup distribution on the performance studied have been discussed in Chapter 7. The conclusion of this work and future direction of research in the area of imaging three phase flow and anaerobic reactors have been discussed in Chapter 8. The appendices discuss the safety related aspects and the operational protocols and data processing methodologies of the DE-DSCT.
Chapter 2

Literature Review

2.0 Introduction

In this chapter a literature review pertinent to the thesis is presented. A critical review of the tomography methodologies for imaging phase holdup distribution of multiphase systems reported in the literature has been presented. A brief review of the studies on the effect of mixing in anaerobic bioreactors has been presented. Since Vesvikar (Vesvikar 2006) has presented a more details review on anaerobic bioreactors it has not been repeated here.

2.1 Overview of Tomography Techniques for Multiphase Flow

Multiphase flow systems are an integral part of the process industry and are employed to carry out chemical and biochemical transformations, and separations. The interaction of the phases in the flow field by extension affects the interaction of the components or species present in the phases that participate in the transformation. Very often the hydrodynamics controls dynamics of the reaction between the chemical species or components involved. Therefore, to determine the performance for any given multiphase system or to enable better design and scale-up, it is important to understand the hydrodynamics of the flow by quantitatively characterizing it. Tomography is a powerful
visualization technique that images domains by sections or by ‘cutting’. It is noninvasive and is not hindered by the opacity of the domain. This ability has made it a powerful research and diagnostic tool for studying multiphase systems.

Tomography techniques applied to image multiphase flow that have been extensively reported in literature can be broadly categorized in terms of the entity used for imaging the domain. These techniques are electrical (including capacitance, resistance and impedance), x-ray and $\gamma$ ray. The transmission of these entities through the domain is a function of the material that constitutes the domain, resistance or obstruction to the transmission could be characterized by a physical property of the material. This physical property is permittivity or conductivity in the electrical based tomography methods and attenuation in the x-ray and $\gamma$ ray based tomography methods. If the domain is represented as small local spatial units called pixels, then the numerical values of these constants for the material in each pixel would collectively provide an image that would depict the spatial distribution of the value of the permittivity, conductivity or attenuation. Each material or phase has a unique value of the constant. This information is used to determine the holdup fraction of each phase in a pixel. The phase holdup images can then be obtained from the physical constant images.

Transmission tomography techniques have been extensively applied to quantify phase holdup distribution in two phase flow systems via imaging such as gas-solid, gas-liquid and liquid-liquid systems. The tomography techniques include: electrical methods (Dong et al. 2005; Gamio et al. 2005; Tortora et al. 2006; Wang 2005; Xie et al. 2006), x-ray (Barouch et al. 2004; Bieberle and Hampel 2006; Hampel et al. 2005; Toye et al. 2001), and single source $\gamma$ ray tomography (Al-Dahhan et al. 2007; Bhusarapu 2005; Khopkar et al. 2005; Kumar 1994; Varma and Al-Dahhan 2007; Vesvikar et al. 2005).
In this section different tomography techniques used to image multiphase flow have been reviewed described briefly. Tomography studies of two phase system have been review in brief. Since this thesis mainly deals with imaging three phase flows, current tomography techniques for three phase systems have been reviewed at length

2.1.1 Electrical tomography

The electrical tomography methods measure the following physical properties across the domain: impedance in the case of electrical impedance tomography (EIT), conductivity in the case of electrical resistance tomography (ERT), and capacitance in the case of electrical capacitance tomography (ECT). These physical properties are measured with the help of sensors that that are located along on the periphery of the flow domain such that they are noninvasive to the flow. The physical properly (capacitance, conductivity or impedance) of the dispersed two phase flow measured across the domain along a line between two sensors is used to determine its local permittivity using a suitable mathematical algorithm. This usually done for small segments of the domain called pixels. The permittivity for each pixel when viewed collectively provides a permittivity image of the flow domain. A constitutive equation that relates the permittivity to the holdup values is used to determine the holdup images from the permittivity images.

The ERT and EIT sensors need to be in contact with the flow at the boundary to make the measurements. These are usually in the form of strip electrodes array flush mounted along the inner side of the vessel along a plane (George et al. 2000; Razzak et al. 2007)at which the sectional phase holdup image is to be determined. This means that the system under study has to be fitted with a cylindrical sensor section with flanges. This section is constructed of an insulated material in which the electrodes are flush mounted.
This fitting doesn’t make this technique truly noninvasive as the system would require physical modifications, however it does remain noninvasive to flow. For ECT this type of modification is not required as the sensors involved need not be in contact with the flow at the boundary. If the systems is a made of metal as in the case of high pressure reactors or columns, then accommodating the sensors posses a engineering challenge.

In electrical tomography methods the number of sensors limits the number of line measurements across the domain. Figure 2-1 shows the arrangement of sensors used for ECT and ERT.

![Figure 2-1: Arrangement of sensors in electrical tomography methods: A.: Electrodes in an electrical impedance tomography system (source: George et al, 2001). B: Sensors used in an electrical capacitance tomography system (source: Warsito and Fan, 2001).](image)

Typically is if \( N \) represents the number of sensors, then

\[
L_N = \frac{N(N - 1)}{2}
\]  

(2.1)

represents the possible number of spatially independent line measurements across the domain (George et al. 2000). Typically one of them is activated as the source with current or charge at any given time and the rest measure the voltage or the capacitance around the
domain. This is repeated till all the sensors have been used as the source and the rest of the sensors have made the measurements. This measurement process can be done very rapidly, often at a time scale that is much smaller than the flow time scale. Hence instantaneous image snapshots can be reconstructed from the data.

The resolution of the image is strongly dependant on the number of spatially independent line measurements possible across the domain. For the system of equation involved in the image reconstruction process, the number of line measurements must be equal to the number of pixels used to reconstruct the image of the domain. The physical dimension of the sensors limits the number that could be accommodated around the domain. At best 12-32 sensors are used (George et al. 2001; Johansen et al. 1996; Tortora et al. 2006; Warsito and Fan 2003), the image reconstruction can be done with 66-496 pixels. These are far too few, hence most of the reconstruction is done with a lot more pixels thereby an underdetermined systems of equations are used. The ratio of pixels to projections of have been reported in the range of 0.3-0.06 have been reported (Nooralahiyan and Hoyle 1997; Tortora et al. 2006; Warsito et al. 1999; Warsito and Fan 2003; Xie et al. 2006). Hence the image reconstruction techniques employ accepted mathematical methods used for an underdetermined system of equations.

Electrical tomography methods usually have a poorer spatial resolution; they provide excellent temporal resolution such that snapshots of the holdup dynamics are could be obtained. If non-iterative algorithms are used to reconstruct the images the phase holdup distribution images could be obtained online. The conductivity or the dielectric constant of the material in the system could at limit the applicability of the system. Short circuiting could occur in the ERT measurements that could distort the true conductivity profile of the domain. Complicated internals in the systems such as draft tubes or heat exchanger tubes
could further limit its use. A major advantage of these methods is that they do not use radioactive substance like in the case of \( \gamma \) ray or x ray tomography, hence health hazards are avoided and stringent operation controls regulatory interventions are not required for their use.

2.1.2 \( \gamma \) ray and x ray tomography

\( \gamma \) ray and x ray tomography methods are based on the measuring the transmission of the \( \gamma \) ray and x ray photons across the flow domain respectively. The physics of transmission process is similar for both types of photons. X ray photons usually have a lower energy than gamma ray photons. The attenuation physics is similar for both hence it is possible to use the same type of detectors to measure the photons of \( \gamma \) ray and x ray. The attenuation of the photons is measured based on the incident photons and those that survive the medium. Attenuation is a function of the composition of the domain, energy of the photons, and the length of the path of the photons through the domain. An x ray tube is used for generating the x ray beam and a radioisotope is used for generating a \( \gamma \) ray beam.

A radio isotope that is used as the source of the \( \gamma \) photons is encapsulated in a steel capsule. This capsule is usually in the order of a few millimeters such that it forms a point source. It is secured by enclosing it in a specially designed collimator device. When this device is shut most of the \( \gamma \) emission is shielded, when it is open such the source is partially exposed, it emits a \( \gamma \) ray beam. This beam could be engineered by the use of collimators to have a cylindrical, conical, or a fan (section of a cone) form. For x ray photons an x ray tube is used which could also be designed to produce a cone, cylinder or a fan beam. The x ray sources when switched off do not emit x ray photons. This however is not possible with \( \gamma \) source as it constantly emits gamma photons which at best could be shielded. Managing
safety related issues with γ sources are more challenging than x ray source. Y ray photons have higher energies ranges than x ray photons which provide greater penetration depths. This is particularly advantageous when it is used for applications related to large scale columns with high attenuating metallic walls.

Independent measurements of photons transmission along a line across the domain between the source and the detector, similar to electrical methods, need to be made for image reconstruction. Different arrangements of source and detectors are possible such as to maximize the number of line measurements across the domain in the least possible time. The first generation tomography systems used parallel beam arrangement where a single source, with a collimated beam and a detector is used. This arrangement is moved laterally to make parallel measurements across the domain in the form of chords as shown in Figure 2-2A.

Figure 2-2: Parallel beam arrangement of source and detectors. A: The source and detector is simultaneously moved at different angular orientation to make parallel line measurements. B: Another arrangement where the source and detectors are fixed and domain is moved laterally at different angular orientation.
An alternate arrangement that would still make measurement of same geometric orientation is shown in Figure 2-2B (Vasquez et al. 2007). The source and the detector are fixed in one position and the domain in moved laterally at different angular positions to make the measurements. The major drawback of parallel beam arrangement is that for a given source position only one line measurement can be made. If a large number of line measurements are required then the process becomes tedious. Parallel beam systems are cost less as a single and detector is used, and the data acquisition system with a single channel is used. To increase the number of line measurement for a given source position a fan beam arrangement of source and detectors is used (Figure 2-3).

![Diagram of fan beam arrangement](image)

**Figure 2-3:** Fan beam arrangement of source and detectors. A: A single source is used with multiple detectors. The fan is rotated around the domain to make the line measurements. B: The source and detectors are fixed and the domain is rotated to make the line measurements.

For a single source position multiple detectors are used in a fan beam arrangement. The γ or x-ray source is collimated such that it emerges as a fan from the point source towards the detector array. The detectors are placed such that they are equiangular and
equidistant from the source. It is also possible to have the detectors placed equiangular in the array with respect to the source along a line instead of along the arch as shown in Figure 2-3A. In the latter arrangement the detectors are not equidistant from the source. The source position and the detector array are reoriented azimuthally along the axis of the flow domain and further line measurements are taken. It is also possible, especially for application where large scale columns need to be scanned, to have the source and detector array fixed and rotate the domain under study (Figure 2-3B)(Toye et al. 2005). For a given number of line measurements of photon transmission across the domain, the fan beam arrangement is able to achieve it faster than the parallel beam arrangement. For scanners that need to generate real time images with high temporal resolution, a fixed scan systems with separate five fan beams that have separate source and equiangular detectors arrays have also been used (Johansen et al. 1996). This help get the maximum possible line measurements in a instant. The transmission data collected along the lines across the domain is processed with a suitable algorithm to obtain a pixel wise local attenuation data. The pixels when viewed collectively provide the sectional attenuation image of the flow domain. A constitutive expression that relates the attenuation of the mixed flow to the phase holdup is used to generate the phase holdup images.

\[ \text{Y ray and x ray tomography techniques do not require contact with flow domain as in the case of ERT. Hence, these techniques are truly noninvasive and don’t require modifications on the system that encompasses the flow domain. The detectors used for such systems are very small, or can be collimated to reduce the volume of spatial line measurements across the domain. A high number of line measurements across the domain are usually possible in these techniques when compared to electrical tomography methods. Hence the spatial resolution is and image accuracy is far superior such that accurate} \]
quantification of phase holdup distribution is possible. However the temporal resolution does not match that of electrical methods. There is a trade off between spatial resolution and accuracy, and temporal resolution as scanners are not capable to collecting the required number of line measurements across the domain at a time scale that is smaller than the time scale of the flow in the system. Electrical methods are also restricted by the properties conductivity and dielectric properties of the material in the flow that could reduced its applicability to different systems. $\gamma$ ray and x ray tomography techniques are not hindered in this manner and therefore are applicable to a wide variety of flow system. $\gamma$ ray tomography is particularly applicable to large flow domains with in high attenuating materials such as metals where x ray tomography is not.

### 2.2 Tomography Studies of Two Phase Flow Systems

Tomography of two phase flows is well established and has been applied to understand multiphase systems pertaining to two phase flow. A close from of equations is for determining the phase holdup image from the permittivity images obtained by electrical tomography methods and attenuation images obtained from radiation $\gamma$ and x ray methods. This closed form of equations includes the physical reality that the sum of the phase holdups values in any given pixel (and even globally) is equal to unity. Tomography studies of the effect of operating conditions and design parameters on various types of two phase flow systems have been exhaustively reported in literature. These include research related to new systems and diagnostics of installed industrial systems. Some examples of its applicability have been discussed here.

Electrical tomography methods have been applied to two phase systems to image phase holdup distribution particularly for applications where the high temporal resolution is
desired, the spatial resolution is usually poor. ECT has been applied to determine the phase holdup images online for gas – oil pressurized flow in horizontal pipes (7.6 cm diameter) at different flow regimes (Gamio et al. 2005; Xie et al. 2006). ERT has been applied to determine the phase holdup distribution and phase velocity using cross correlation between ERT measurements at two planes for air – water flow in vertical pipes(Dong et al. 2005). ERT has also been used to determine the phase holdup images in liquid – solid, and air – liquid flows in pipes (Wang 2005). For the application of ERT it is important that the continuous phase in the flow is a conductor and not an insulator, hence it is not possible to use it for gas – solid applications where the gas is continuous. EIT has been used in a gas – solid riser (14 cm diameter and 5.7 m height) to determine the solids phase holdup distribution profiles (Tortora et al. 2006). All the electrical tomography applications mentioned have good temporal resolution and could be used for applications where online results have a premium over high accuracy in quantification.

X ray computed tomography (CT) is not limited by the conductivity and capacitance characteristics of the flow domain. X photons usually have a lower energy than \( \gamma \) photons and hence are useful for applications where the density contrast between two phases is poor. It usually can’t be used for large scale or high attenuating flow system as the low energy x ray photons have a poor penetration depth. This leads to a poor signal to noise ratio in the transmission data collected by the detectors. X ray CT has been applied to study phase holdup distribution and flow structure in various two phase systems. The structure and dynamics of cloud cavitation in venturimeters with gas – liquid flow has been studied (Barouch et al. 2004). Phase holdup distribution studies of gas – liquid flow horizontal pipes (Bieberle and Hampel 2006) have been reported. X ray CT has also been applied systems
where the solid phase is static and the flow is two phase consisting of gas – liquid flow alone like columns with structured packing (Aferka et al. 2007; Toye et al. 2001) and packed distillation (Toye et al. 2005; Toye et al. 1996).

$\gamma$ ray tomography has a wide range of applicability due to its true noninvasive nature, its ability to scan large scale system with high attenuating walls, and like x ray it is unhindered by the conductivity and capacitance limitations of the system. At the Chemical Reaction Engineering Laboratory (CREL) single source $\gamma$ ray computed tomography (CT) systems with a fan beam arrangement of source and detectors were developed by Kumar (Kumar 1994) and Roy (Roy 2006a). Various studies of two phase systems using $\gamma$ ray CT have been reported in literature a significant number of these are from CREL. The effect of operating and design parameters such as: superficial gas velocity, column diameter, distributor design and surface tension on the phase holdup distribution have been studied in a gas – liquid (air – water) bubble column (Kumar and Duduković 1997). The solids holdup has been studied in a solid-liquid circulating fluidized (Roy et al. 1997) and a gas-solid riser (Bhusarapu 2005) as a function of superficial liquid and gas velocity respectively. Gas holdup distribution had been studied in large scale (45 cm diameter) gas-liquid (air – water and air – drake oil) bubble column (Chen et al. 1998). Gas holdup distribution studies have been done in gas – liquid stirred tank systems with impellers (Hampel et al. 2007b; Khopkar et al. 2005; Rammohan 2002). The effects of superficial gas velocity and sparger design on gas holdup have been studied in cold flow biological systems such as photobioreactor for producing algae (Luo 2005a) and anaerobic biodigesters for methane generation (Karim et al. 2004; Varma and Al-Dahhan 2007).
Single source γ ray CT (like some examples of x ray CT) can also be used for phase holdup studies in three phase systems where the solid phase is stationary. In such three phase system the flow is usually gas – liquid, hence it first studied as two phase system with gas – solid and liquid – solid to determine the solid phase holdup. The solids phase holdup remains constant when the system is operated with gas – liquid – solid, therefore the unknowns are still the holdups of the two phases that are in the flow. This could be solved as a two phase system to determine the phase holdup distribution. This approach has been used to determine the effect of operating conditions such as liquid flow rate and superficial gas velocity on the gas and liquid phase holdups in monolith reactors (Roy and Al-Dahhan 2005) and packed column with structured packing (Roy et al. 2004; Schubert et al.). Phase holdup studies have also been done in ebullated bed column where the gas – liquid flow co-current from the bottom to the top through a packed bed of solids (Chen et al. 2001).

However, a significant number of chemical and biochemical systems, consist of three dynamic phases (gas – liquid – solids). Tomography methods discussed thus far cannot be applied to multiphase phase systems with three dynamic phases. This due to a equation closure issues that arises (discussed in the next section) when single source (energy) x ray and γ ray CT, or ECT, EIT and ERT are used. Unless there is some prior information about the holdup is available, like the holdup of the solid phase in structured packing, it is not possible to use these techniques to image three phase systems.

2.3 Tomography of Three Phase Flow Systems

Chemical and biochemical reaction systems often involve solids that facilitate the transformation. There are usually three phases involved in such systems that often dynamic.
The principles of tomography techniques widely applied to study various two phase systems could also be utilized to understand three phase flow. A major issue is solve a set of equations to determine the phase holdup from the tomography image data. This section the efficacy and limitations of tomography techniques reported in literature for imaging three flows have been discussed.

Line measurements across the domain are used to generate images with pixels that represent numerical values of a physical property of the dispersed flow. This physical property is attenuation for γ and x ray CT and permittivity or capacitance in case of EIT and ECT. In a generic form (2-2) represents a constitutive equation that related attenuation of the flow $\mu_{GLS}$ to the holdup fractions $\varepsilon_G$, $\varepsilon_L$, and $\varepsilon_S$ of gas, liquid, and solid respectively.

$$\mu_{GLS} = f_s(\varepsilon_G,\varepsilon_L,\varepsilon_S)$$  \hspace{1cm} (2-2)

In (2-2) $\mu_{GLS}$ is the known and $\varepsilon_G$, $\varepsilon_L$, and $\varepsilon_S$ are the unknown quantities that have to be determined. A second equation could be obtained from the universal fact that

$$\varepsilon_G + \varepsilon_L + \varepsilon_S = 1,$$  \hspace{1cm} (2-3)

as holdups are fractions. In a two phase flow system it could be solve right away to determine the holdup as (2-2) would have two unknowns and (2-3) would just two holdup terms. For equation closure a second constitutive equation is required that related holdup to some known parameter. This second equation could be obtained by scanning the system with a photon of different energy. As attenuation is a function of energy, this equation would also relate the same holdup values to a known attenuation coefficient based on a different energy. This way a closed set of equations could be solved to determine the three unknown phase holdup values.
In electrical tomography methods a generic form of the constructive expression that would relate conductivity or permittivity, both represented as \( \sigma_{GLS} \) in (2-4) for the sake of this discussion, to holdups \( \varepsilon_g, \varepsilon_L, \) and \( \varepsilon_s \).

\[
\sigma_{GLS} = f_1(\varepsilon_g, \varepsilon_L, \varepsilon_s) \quad (2-4)
\]

A second equation of a similar form is required along with (2-3) for equation closure. Some prior information or data could also be used. In multimodality tomography methods that use \( \gamma \) ray CT with ERT or ERT, (2-2), (2-3) and (2-4) could used to get a closed form of equation to determine the phase holdups. Based on this discussion the techniques reported in literature for determining phase holdup distribution in three phase systems could be classified as electrical tomography, dual energy \( \gamma \) and x ray tomography, and multimodal tomography.

### 2.3.1 Electrical Tomography

Electrical capacitance (ECT) and resistance (ERT) tomography methods have been applied to three phase floe systems. ECT measurements provide the capacitance data between sensor pairs, if \( C_i \) represents the measured capacitance between the sensor pair then, capacitance is related to permittivity as (Warsito and Fan 2005):

\[
C_i = \frac{1}{V_i} \int_{A_i} (\epsilon(x, y, z) \nabla \phi(x, y, z)) dA_i, \quad (2-5)
\]

where \( \epsilon(x, y, z) \) is the permittivity distribution, \( \phi(x, y, z) \) is the electrical field distribution, \( V_i \) is the voltage difference between the electrode pair. The objective of the image processing algorithm is to determine the values of \( \epsilon(x, y, z) \) for \( x, y \) and \( z \) based on (2-5). Image processing has been carried out using liner back projection (LPB) with appropriate
filters. More advanced algorithms have been reported like neural networks (Nooralahiyan and Hoyle 1997) and a neural network based multi-criterion optimization image reconstruction technique (NN-MOIRT) proposed by Warsito and Fan (Warsito and Fan 2001).

ERT measurements use direct current to determine the conductivity and data between two electrode pairs. If $I$ represents the input current (or current injection) and the measured current is represented as $I_i$ at the sensor $i$, then

$$\int_{S} \sigma \frac{\partial \phi}{\partial n} dS = +I, \quad (2-6a)$$

represents the applied current and

$$\int_{S} \sigma \frac{\partial \phi}{\partial n} dS = -I_i, \quad (2-6b)$$

represents the sensed current at the electrodes (Dong et al. 2006). The relationship between electric potential distribution and conductivity is given by Laplace equation as (Dong et al. 2006; George et al. 2000):

$$\nabla \cdot (\sigma \nabla \phi) = 0. \quad (2-7)$$

The measured conductivity data given by (2-6) can be though of as boundary conditions for (2-7). Equation (2-7) is solved using finite deference methods to determine the conductivity map of the domain. Tortora et al (Patel et al. 2007) have reported many from literature that related conductivity to holdup, these include: series and parallel
 impedance models, the Maxwell mixture model, and Bottcher model, Beck model and the Rayleigh model.

The Maxwell-Hewitt relation is most commonly used to relate the conductivity to the of the insulated phase holdup values {George, 2001 #8; George, 2000 #94:

\[ \frac{\sigma}{\sigma_L} = \frac{1 - \varepsilon_j}{1 + \alpha \varepsilon_j}. \]  

(2.8)

Where \( \varepsilon_j \) is the holdup of the insulated phases, liquid and gas, hence

\[ \varepsilon_j = \varepsilon_c + \varepsilon_s. \]  

(2-9)

As a logical extension of (2-3) \( \varepsilon_j \) as can be represented as

\[ \varepsilon_j = 1 - \varepsilon_L. \]  

(2-10)

The value for the parameter \( \alpha \) is chosen based of the size of the insulator inclusion is the conducting media. The values of 1/2 have been used for three phase gas – liquid – solids flow and 3/5 for gas – liquid flow have been suggested {George, 2001 #8; George, 2000 #94; Razzak, 2007 #93}.

EIT measurements use alternating current to determine the impedance sensors on the periphery of the flow domain. Equation (2-7) is solved to determine the conductivity. It assumed that the capacitance contribution to impedance is minimal as the sensors are used are usually have s small surface area. The true direct current conductivity can be determined
based on the frequency of the current and the permittivity of the continuous phase (George et al. 2001).

A brief summary of the published tomography studies based on electrical methods has been described as follows:

(Warsito and Fan 2003; Warsito and Fan 2005)

Wasito and Fan (Warsito and Fan 2003; Warsito and Fan 2005) have studied the phase holdup distribution in a slurry bubble column (gas – liquid – solid) system with high temporal resolution using ECT. A three phase capacitance model is proposed based on two region model to relate the permittivity to holdup. This two region model assumes at any given time the solids concentration in liquid – solids emulsion is constant throughout the cross section at which the system is scanned. A series and parallel capacitance models were used to related permittivity (determined from capacitance measurements) to holdups represented as:

\[
\frac{1}{\varepsilon_{GLS,t}} = \frac{\varepsilon_G}{\varepsilon} + \frac{(1-\varepsilon_G)}{\varepsilon_{LS}}, \quad \text{and} \quad \quad (2-11a)
\]

\[
\varepsilon_{GLS,p} = \varepsilon_G \varepsilon_{G} + \varepsilon_{LS} (1-\varepsilon_G), \quad \quad (2-11b)
\]

respectively. Where \(\varepsilon_{GLS}\) and \(\varepsilon_{LS}\) are the permittivity of the gas – liquid – solid flow and the liquid – solid emulsion respectively. The permittivity \(\varepsilon_{LS}\) is further expressed as:

\[
\frac{1}{\varepsilon_{LS,t}} = \frac{\varepsilon_{SE}}{\varepsilon} + \frac{(1-\varepsilon_{SE})}{\varepsilon_{LS}}, \quad \text{and} \quad \quad (2-12a)
\]
Here $\varepsilon_s$ is the dielectric constant of the solid. Based on (2-11) and (2-12) the authors proposed the following series and parallel equation for gas holdup in the system:

\[
\varepsilon_{G,s} = \frac{1 - \gamma \hat{\varepsilon}_{GLS}}{\hat{\varepsilon}_{GLS} (k_{Ls} - 1) + 1}, \quad \text{and}
\]

\[
\varepsilon_{G,p} = 1 - \gamma \hat{\varepsilon}_{GLS}.
\]

Here $\hat{\varepsilon}_{GLS}$ is the reconstructed normalized permittivity of the three phase flow, $k_{Ls}$ is the permittivity ratio of the liquid – solid packed bed to gas and $\gamma$ is defined as:

\[
\gamma = \frac{k_{Ls} - 1}{k_{LS} - 1},
\]

(2-14)

where $k_{LS}$ is the permittivity ratio of the liquid – solid part of the flow to gas phase. The phase holdup values were obtained using the following expression:

\[
\varepsilon_G = \beta_s \varepsilon_{G,s} + \beta_p \varepsilon_{G,p} + \beta_g \varepsilon_{G,g} + \beta_{pp} \varepsilon_{G,pp}, \quad \text{where}
\]

\[
\beta_s + \beta_p + \beta_g + \beta_{pp} = 1.
\]

(2-15)

The subscripts $s$ and $p$ denotes series and parallel capacitance connection liquid – solid phase in the emulsion for the first subscript in (2-15) and gas – emulsion phase for the second subscripts. The $\beta$ terms represent the weighted coefficients of these interactions. In other words it determines the about of the phases interacting in a series or a parallel manner. The solids holdup was found using
\[ \varepsilon_s = \varepsilon_{SE} (1 - \varepsilon_G). \]  

(2-16)

The value of \( \varepsilon_{SE} \) in the emulsion is calculated based on the tomography image from the no-bubble region assuming two phase flow of liquid – solid. Equations (2-15) and (2-16) are used to determine the gas and solid holdups respectively, the liquid holdup can be determined using (2-3). The authors do not provide first principle based justification for the values of \( \beta \). The instantaneous values of the fraction of series – series and series – parallel, etc. interaction of the phases would be hard to obtain. There no way to confirming the validity of the two region model in terms of operating regime of the system. It is at best an assumption which could be valid at high solids loading in the system, or at low gas flow rates. Obviously at high has flow rates the gas would displace the sold and liquid in manner that the solids holdup in the liquid would not be the same in the gas free region of the column.

(Razzak et al. 2007)

Razzak, Baeghi and Zhu (Razzak et al. 2007) used ERT to study the phase holdup distribution in a gas – liquid – solid circulating fluidized bed 7.6 cm in diameter and 5.97 m tall. The azimuthally averaged conductivity data is related to hold up of the insulating phase (gas – solid) using a form of (2-8). For equation closure the pressure drop measurements were made close to the scanning cross section. The pressure drop is related to holdup as presented as:

\[ \frac{\Delta P}{\Delta Z} = \rho \varepsilon_g g = (\varepsilon_G \rho_c + \varepsilon_s \rho_L + \varepsilon_s \rho_s) g, \]  

(2-17)
where $\Delta P$ is the pressure drop at the measured section and $\Delta Z$ is the height of the measured section. The gas density was assumed to be negligible and the solids holdup was calculated as

$$\varepsilon_s = \left( \frac{\Delta P}{\Delta Z} - \varepsilon_l \rho_l \right) \rho_s.$$

Equations (2-8), (2-18) and (2-3) provide a closed form of equation to determine the holdups of gas, liquid and solid.

This approach doesn’t provide images of phase holdup distribution it provides radial profile that assume azimuthal symmetry. The pressure drop measurements are not local (pixel wise) nor are they radial. The solids holdup values obtained based on it is a global value at the cross section. Hence, this approach doesn’t necessarily provide the complete picture of the flow, particularly if it is applied to systems with a larger diameter.

### 2.3.2 X Ray Tomography and Y Ray Tomography

The transmission data measurement along the line is processed to obtain a pixel wise attenuation image. The constitutive equation that related the attenuation values of the flow to the holdup is a linear sum of the product of the holdup and the attenuation of the phases. This is represented as:

$$\mu_{GLS} = \mu_G \varepsilon_G + \mu_L \varepsilon_L + \mu_S \varepsilon_S,$$

where $\mu_G$, $\mu_L$ and $\mu_S$ are the attenuation values of the gas, liquid and solids respectively. The equation is the same for the attenuation values generated by $\gamma$ and x ray CT. A second
equation is obtained in the form of (2-19) is obtained when the system is scanned with a photon of different energy. These two equations along with (2-3) provide a closed set of equations when applied to each pixel in the domain provide the images of the phase holdup distribution. All the discussion related to three phase studies related to γ and x ray CT evolve around this set of equations. A brief compendium of the published work is as follows:

(Gehrke and Wirth 2005)

Gehrke and Wirth (Gehrke and Wirth 2005) have applied dual energy x ray CT to determine phase holdup distribution images in a gas – liquid – solid systems with gas as the continuous phase in a fluidized bed and the liquid as the continuous phase in a slurry bubble column. Attenuation images were generated processing the transmission data from x ray tube setting at 80 keV and 140 keV using algebraic reconstruction techniques (ART). A Plexiglas column with 10 cm diameter and a height of 1.8 m was used for the study. Since the x rays photons are of lower energy the authors suggest scanning columns with a higher diameters, especially with continuous liquid phase would not be possible.

(Hu et al. 2005)

Hu et al (Hu et al. 2005) have developed a dual energy x ray CT which uses a 160 keV x ray source. The polyenergetic nature of the x ray spectra has been exploited to generate x ray photons of different energies. A filter wheel consisting of a copper, lead and an air section is used with the x ray source. The copper part blocks the lower energy photons and allows only the higher energy photons to pass through; this is referred to as hard beam. The air part of the filter allows all the photons to pass through; this is referred to as the soft
beam. This way the two attenuations with the hard and soft beam are obtained respectively
to obtained equations along the lines of (2-19).

Two fixed fan beams are used for this study, hence the number of line measurements
available are limited. An ART based algorithm was developed to generate the image. Due to
the limited line measurements the images generated appear to be distorted. This system was
applied to study gas – liquid – liquid (air – water – oil) flow in a horizontal pipe of 10 cm
diameter.

(Froystein et al. 2005)

Froystein et al (Froystein et al. 2005) have developed dual energy $\gamma$ ray CT system
with a parallel source detector arrangement using a single detector. The 31 keV and 81 keV $\gamma$
photo peaks from a $^{133}$Ba isotope of 30 mCi strength was used to generate the attenuation
images. Line measurement data was collected for 12 source angles, for each source angle 20
parallel measurements were collected. Test phantoms 7.6 cm in diameter representing
different spatial configurations of gas – liquid – liquid (air – water – oil) flow were used for
validation.

The $\gamma$ photons used in this study have a very low energy hence are easily attenuated.
To prevent attenuation by the wall of the pipe, the authors used a pipe made of special low
attenuation polymer. The number of line measurements across the domain is less than the
number of pixels. Hence and underdetermined systems of equations was used.

(Nikitidis et al. 1999; Nikitidis et al. 1998)
Nikitidis et al. (Nikitidis et al. 1999; Nikitidis et al. 1998) studied gas-solid-solid flow consisting of a binary mixture of solids in a 14.4 cm diameter hopper with a conical bottom. A dual energy γ ray CT system consisting of a fan beam arrangement of source and detectors was used. The 44 keV and the 100 keV γ photo peaks from a $^{153}$Gd source were used. Source strength of 1 Ci was used to compensate for the low energy of the γ photo peaks from the $^{153}$Gd source.

(Rados 2003a; Rados et al. 2005a)

Rados et al. (Rados et al. 2005a) and Rados (Rados 2003a) have used a single source γ ray CT to determine the phase holdup distribution in gas-liquid-solid system. This could be represented as (2-19). In addition pressure sensors were used to determine the pressure drop at the level of the scan which provided an additional equation in the form of (2-18). Using the assumption that the solid holdup remains constant in the liquid-solid region and the gas-liquid-solid region at a given cross section and (2-3) closed system of equation is available to solve for the holdup of the three phases. As in the case of Razzak et al. (Razzak et al. 2007) this methods uses a global solids holdup value at a given section in the system to determine the local gas holdup at that same section.

### 2.3.3 Multimodality Methods

Multimodality methods integrate two different forms of tomography to image phase holdup distribution in three phase flow. The objective is get two separate constitutive equations that related a intrinsic properties to holdup, these equation when used with (2-3) from a closed set of equations to determine holdups. Measurements different tomography systems are carried out at the same cross section of the flow system. γ ray CT and ECT have
been combined (Johansen et al. 1996) to use the respective constitutive equations from these two techniques for equation closure. Other tomography systems such as two single energy γ ray and x ray CT systems (Hale 2007; Rapaport et al. 1995), EIT and γ ray densitometry (George et al. 2001) have been used as multimodality systems. A brief summary of the multimodality methods reported in literature have been summarized as follows:

(Johansen et al. 1996)

Johansen et al (1996) developed a fixed γ ray CT system that uses five fixed fan beans with 17 detectors each. An ECT system was used separately at the same cross section of the flow system to determine phase holdup distribution. The system was developed for a domain size of 10 cm and was used for determining phase holdup distribution in gas – liquid – liquid flow pertaining to the air – oil – water flow in horizontal pipes.

This system has limited number of line measurements across the domain and reconstructs the images using higher number of pixels than the measurements. Different image reconstruction grid orientations were used for the two techniques. Due to this some inconsistencies in the pixel to pixel mapping of the images appear that affect the phase holdup images.

(Hjertaker et al. 2005)

Hjertaker et al (Hjertaker et al. 2005) developed the multimodality system of (Johansen et al. 1996) further to over come some of the salinity related challenges that effect the ECT measurements of air – oil – water flow. ECT requires that the continuous phase in the flow to be nonconductive. The salinity in water drastically changes its attenuation and
there by introduces errors. A high frequency magnetic filed water fraction meter was included in the system to determine the fraction of water in the system. This information was used along with the ECT and γ ray CT system to image the phase holdup distribution in the flow.

(George et al. 2001)

George et al (George et al. 2001) have developed a multimodality system the combines EIT with γ ray densitometry. This system was applied to study gas – liquid – solid flow in 20 cm diameter slurry bubble column. This multimodality method only provides a radial holdup profile of the phases in the system, as densitometry provides the radial data. Hence phase holdup images were not reconstructed and azimuthal symmetry in the flow was assumed. The permittivity data obtained from EIT was fitted into a mathematical expression relating the conductivity to radial location for comparison with the γ ray densitometry data. A constitutive equations in the form of (2-8) relating radial conductivity to phase holdup, and in the form of (2-19) relating radial attenuation to phase holdup were used with (2-3) to determine the radial phase holdup profiles.

(Rapaport et al. 1995)

Rapaport et al (Rapaport et al. 1995) have integrated a γ ray source and an x ray source to make a dual energy tomography system. Different types of detectors were used for γ and x photons. The 661 keV γ photons from a 2.2 Ci $^{137}$Cs source and the 160 keV x photons were used. A fixed source detector arrangement similar to that shown in Figure 2-3B was used. Domains up to 40 cm could be imaged with this scanner.
The authors have used this scanner for nondestructive testing applications pertaining to structures related to the aircraft and the chemical industry. The objective was use the different contrasts available with the lower energy x-ray images and the higher energy γ-ray images. Although the authors have not reported any multiphase flow studies with this system, it could perfectly be used for such applications using the approach of a dual energy tomography.

2.4 Remarks on Tomography of Three Phase Systems

Tomography of two phase flow, particularly x-ray and γ-ray based tomography has reached a level of maturity in terms of the confidence in the data obtained for research and diagnostic applications. It has been applied exhaustively to study different types of reactors and process equipments dealing with two phase flow. In contrast, tomography of three phase flow system has not evolved to the same extent. One of the early reviews on the three phase flow problem of gas – oil – water flow in the oil industry by Thorn et al (Thorn et al. 1997) looked at methods where the phases were separated from the flow to determine their holdup fractions and suggestion were made for possible methods for deriving online measurement systems based on electrical and γ methods. One of the initial works on a tomography systems for three phase flow was reported by Johansen et al (Johansen et al. 1996) The authors proposed a multimodality method for imaging. All the techniques developed and reported since have had limitations related to either the domain size, or the weak fundamentals of the assumptions relied up for the results, or resolutions limitations that have collectively restricted the applicability of the technique and have prevented its use for research and diagnosis as ubiquitously as tomography systems for two phase flow.
The single modality based methods used which include ECT (Warsito and Fan 2003), ERT (Razzak et al. 2007) and γ ray CT (Rados 2003a; Rados et al. 2005b) for imaging three phase flow rely on assumptions such as local solids holdup is equal to global solids holdup at the a given section. Also pseudo two phase assumptions where the solid and liquid are assumed form a uniform emulsion comparable to a single phase might be valid for a system with high solids loading used at certain operating conditions. These assumptions could be considered valid for very limited types of systems at operating conditions and cannot be applied universally to image phase holdup in three phase flow.

The constitutive equations that electrical methods use which relate holdup to conductivity and permittivity, for three phase or two phase flow, are dependent on parameters (equations (2-8), (2-14) and (2-15) (Dong et al. 2006; Gamio et al. 2005; George et al. 2001; Razzak et al. 2007; Warsito and Fan 2003) whose values vary based on the system geometry and flow regimes. Although authors provide justifications for the values chosen or even have validated them with the use of established techniques (George et al. 2000) these still would introduce uncertainties in the results or would require certain degree of assumptions when applied to different systems. Also, the electrical methods always used a underdetermine system of equations for image constructions due to the physical size limitation of the sensors that are used. This compromises the image quality in terms its spatial resolution and numerical accuracy. For most of these scanners there is tough choice to be made in terms of spatial resolution and temporal resolution. When such system are used in multimodality methods the limitation are carried over and are further compounded with limitations of the other method applied. Moreover multimodality methods require
investment in two fundamentally different platforms of sensors and data acquisition systems thereby increasing cost.

In contrast to electrical methods γ and x ray methods have a simpler and linear constitutive equation that relates the attenuation coefficient of the flow to the holdups of the phases (2-19). This enables its application to various flow systems without dependency on the type of flow regime or geometry of the system. Dual energy γ and x ray CT have been developed and applied to three phase flow on its basis. A common running theme in all these methods sighted in the literature is the poor penetration depth of the photons due to their low energy. Hence, applications dual energy γ ray and x ray CT discussed are limited to small domains from 5-7.5 cm, and require special modifications to reduce the attenuation by the walls at the section of the flow system where the scans are to be conducted. This has led to use of some high strengths of the γ source (in the order of 1000 mCi) (Nikitidis et al. 1999; Rapaport et al. 1995) to be used which bring in serious safety concerns, or they require long scanning times to get an appropriate counts data. Long scanning time has also been reported because of the use a single ray parallel arrangement of source and detectors (Froystein et al. 2005). The long scanning times may limit the number of data line measurements and thereby affect the image quality.

Based on the review of reported literature, it can be concluded that the research efforts towards developing techniques for imaging phase holdup distribution in three phase flow at best have met with partial success. The limitations have largely been in terms of; its applicability very small scale of systems with limited research applications, accuracy and reliability of the data obtained, and property and regime of flow. Since the objective of using the tomography techniques in research is to be able to get accurate phase holdup distribution
information that is as close as possible to the truth. Using techniques with limitations or assumption would hinder that objective. There is a strong motivation to develop a robust tomography technique for imaging three phase flow systems such that; it is not limited by the type of material in the flow or construction the system, it is truly noninvasive, it can image large scale flow systems, and doesn’t rely on assumption related to the flow or depend on some prior information related to the flow. This is imperative when tomography data is used to develop and benchmark mathematical models and correlations and closures for computational fluid dynamics (CFD) studies of multiphase flows.

2.5 Selection of tomography method for three phase flow

A critical review of the literature for techniques discussed for the tomography of three phase flow indicates that $\gamma$ based CT methods are generally more advantageous than electrical methods. This is true for high temporal imaging systems (Johansen 2005; Johansen et al. 1996) where the line measurements available are limited. Since the need for equation closure requires that two independent constitutive equations are available, a dual energy $\gamma$ ray CT would be a favorable choice. This thesis considers the design and development of a dual energy $\gamma$ ray CT system for imaging three phase flow. As discussed earlier, $\gamma$ ray methods in general find favor over electrical methods as:

- The constructive equation relating phase holdup to attenuation coefficient of the flow in independent of parameters and in simple.
• More line measurements are possible such that high spatial resolution images can be generated.
• The techniques is truly noninvasive, hence no modifications on the flow systems are required.
• Restrictions in the applicability due to the dielectric constants or conductivity of the continuous phase, typical in electrical methods, do not exist with γ ray CT. Hence the system could be applied to a wide range of multiphase flow systems.

To increase its applicability to image phase holdup distribution in large scale systems the choice of γ photons sources (isotopes) should be such that sufficient penetration depths are available. As attenuation is an inverse nonlinear function of the energy γ photons, at higher energies (above 150 keV) there should be a reasonable number of photons that survive the domain and be detected with minimal noise. This could be accomplished with lower energy photons (less than 150 keV). However this would require really high source strengths (concentrations) that would create radiation hazards (Nikitidis et al. 1999; Nikitidis et al. 1998) and would provide only incremental returns in terms of the counts detected. Therefore the choice of two higher energy γ photons for dual energy γ ray CT is more suitable if it is to be designed for versatility in terms of: the domain size, high attenuating material in flow and system structure, and low source strengths.

2.6 Anaerobic bioreactors for methane generation

The rapid of expansion of the livestock industry in the United States from an environmental perspective has created both, opportunities and problems. The improper disposal of the large quantity of manures generated at dairy, swine, and poultry and other
animal farms have created environmental concerns such as contamination of water bodies and foul odor nuisance. The decomposing manure waste from livestock in water leads to a high biological oxygen demand (BOD), increases the pathogens, and lead to the emission of green house gasses such as methane. Methane has a higher green house potential than carbon dioxide.

Viewed more positively, the waste provides an opportunity for the generation of methane as renewable source of bioenergy by anaerobic digestion of the waste. Aerobic biological fermentation, or anaerobic digestion, offers several advantages over other waste treatment processes as it converts odor-causing components and other components in the organic matter to methane and carbon-dioxide, which are odorless. Odor control is one of the main concerns associated with livestock farms.

With better treatment of the waste with the use of anaerobic bioreactors (ABs) methane could be used a clean source of energy. Anaerobic digestion produces biogas with methane contents anywhere between 55% - 80% and the rest is mainly carbon dioxide (Lusk 1998). The end solid product could be used as soil conditioner for land filling purposes or it could be compost and used as fertilizer for agricultural applications.

The degradation of organic waste by the process of anaerobic digestion is bought about by a complex ecosystem of microorganisms (Demirer and Chen 2004). The long chain organic molecules in the waste are first hydrolyzed into simple soluble organics. These organic are converted into long chain fatty acids. The acids are further decomposed into acetate which is finally converted into methane and carbon dioxide. All these conversions are carried out by different class of microorganisms that form the part of an ecosystem for
which an anaerobic environment is necessary. As Vesvikar (Vesvikar 2006) and Hoffman (Hoffmann 2005) have reviewed this the anaerobic digestion process at length, it has not been repeated here. The anaerobic bioreactor facilitates the degradation process by providing the appropriate global conditions that facilitate the generation of methane optimally.

Many configurations of anaerobic bioreactors have been tested and employed in lab scale and large scale systems. In a Department of Energy sponsored study by Lusk (Lusk 1998), which covered about a hundred field digesters consisting of mixed, plug flow, and lagoon digesters, it was reported that 70%, 63% and 22% of them respectively failed. Inadequate mixing and poor design of the digester were identified as the major reasons. Mixing has other important functional benefits in anaerobic digesters, such as aiding the process of venting the biogas generated, providing efficient utilization of the entire digester volume, dispersing organisms and metabolic end products and preventing foaming and scum formation. However, neither quantification of the digester’s flow behavior, nor the effects of mixing on the performance of anaerobic digesters are well understood or reported in the literature.

Research on the effects of mixing on anaerobic systems have produced contradictory results (Angenent and Sung 2001; Angenent et al. 2002; Hashimoto 1982; Ho and Tan 1985; Hoffmann 2005; Karim et al. 2005a; Karim et al. 2005b; Rivard et al. 1995; Smith et al. 1996). The results do not elucidate or quantify the effects of mixing on the performance in a structured manner. Multiple configurations for mixing have been utilized for anaerobic bioreactors. These can broadly be grouped as mechanical agitators (impellers and turbines), slurry (liquid) recirculation, and biogas recirculation (also called gas-lift digesters). Mechanical mixers in general are most effective for mixing but require high power input and have
moving parts. Slurry recirculation anaerobic bioreactors also require high energy input, and they cause operational difficulties as the slurry needs to be continuously pumped. The high shear generated in systems with mechanical agitation and slurry recirculation may not be favorable to the microorganisms. Hence less energy intensive options such as gas-lift systems with no moving parts, find favor over mechanically agitated and slurry recirculation systems (Kontandt and Roediger 1977; Lee et al. 1995).

In gas-lift digesters, part of the biogas generated is recirculated with the aid of blowers. The gas is usually restricted to a particular region of the reactor which is enclosed by a draft-tube. The gas reduces the average density of the medium in the draft-tube, which creates a density gradient in the regions of the draft tube and the rest of the reactor. This density gradient induces buoyancy forces which cause the liquid/slurry to circulate and mix the system. Hence, since gas directly affects the hydrodynamics of the anaerobic bioreactors, it is important to fundamentally understand and quantitatively assess the effects of the gas holdup distribution in the draft tube on the hydrodynamics and the mixing intensity of the digester. Such investigations have not been reported in literature. Gas-lift anaerobic bioreactors have a low length to diameter (L/D) ratio, compared to the air-lift bioreactors used in aerobic systems. A detailed review of the modes of mixing and different configurations of anaerobic bioreactors available for anaerobic digestion, and the particular advantages of gas-lift bioreactors has been varied out by Vesvikar (Vesvikar 2006). Hence it has not been repeated here. Vesvikar’s (Vesvikar 2006) doctoral thesis focuses on the use of radioactive particle tracking and computational fluid dynamics (CFD) to investigate the effect of various operating and design parameters on the liquid/slurry circulation pattern in gas-lift lab and pilot scale anaerobic bioreactors.
Gas-lift anaerobic bioreactors are highly opaque three phase systems due the nature of the wastes treated, the dense color of the biomass, the dispersed gas, and the walls of the digester. Therefore, noninvasive measurement techniques are needed to properly characterize their hydrodynamics and mixing. X-ray computed tomography (CT) and computer automated radioactive particle tracking (CARPT) are powerful tools that could help study the hydrodynamics of such systems. Both techniques can help in understand the hydrodynamics and also help design such system for optimal methane generation.
Chapter 3

Algorithm for Image Reconstruction: A Comparison of Expectation Maximization and Alternating Minimization

3.0 Introduction

Image reconstruction is a key part of the tomography process. The transmission data of $\gamma$ photons, representing the line integrals of the attenuation along a path between the source and detectors across the domain, are processed mathematically to obtain an image. The image is a collection of pixels such that each pixel represents a small spatial segment of the domain. The liner attenuation coefficients values of each pixel in the domain when view collectively represents the attenuation image of the domain. Various mathematical techniques have been proposed for the image reconstruction problem. Several authors have made contributions to the theory of reconstructive tomography and an early overview of such algorithms was presented by Snyder and Cox (Snyder and Cox 1977).

Algebraic algorithms such as Fourier/convolution techniques when applied for image reconstruction either assume the systems to be azimuthally symmetric in distribution or consider the $\gamma$ ray transmission process to be deterministic there by completely ignore the stochastic nature of the data (Bieberle and Hampel 2006). Expectation maximization (Lange and Carson 1984) and Alternating minimization (O'Sullivan and Benac 2007) are amongst algorithms account for the stochastic nature of the $\gamma$ ray transmission across the
domain of interest. This makes these algorithms more favorable for image reconstruction to
determine the phase holdup distribution.

The EM algorithm has used extensively to process $\gamma$ ray CT data to image holdup
distribution in various two phase systems (Karim et al. 2004; Kumar et al. 1995; Kumar et al.
1997; Patel et al. 2007; Rammohan 2002; Roy and Al-Dahhan 2005; Roy et al. 1997; Roy et
al. 2004). Some of its applications have been discussed in Chapter 2. Studies have been
conducted in the past to compare filter back projection (FBP), radon transform, and
algebraic reconstruction techniques (ART) (Kumar 1994) at the chemical reaction
engineering laboratory (CREL). More recently a similar comparative study has been done
from transmission data from a parallel and fan beam scanner with limit projections (Patel et
al. 2007). Both these have concluded that the EM algorithm performs far batter that the
others it was compared to. Hence these have not been repeated here.

In this study the Alternating Minimization algorithm proposed for transmission
tomography by O'Sullivan and Benac (O'Sullivan and Benac 2007) has been applied to the
case of single energy $\gamma$ ray CT for imaging phase holdup distribution in two phase systems.
The AM and the EM algorithms have been compared. Phantoms used for this study are
designed to represent two phase systems studied with EM in the past. The holdup
distribution results obtained with the AM algorithm are quantitatively compared with those
obtained with the Expectation Maximization (EM) algorithm as proposed by Lange and
Carson (Lange and Carson 1984), for simulated $\gamma$ ray transmission data from phantoms. In
addition, the holdup distribution images using the filter back projection (FBP) method as
proposed by Kak and Slaney (Kak and Slaney 1998) have been included for qualitative
comparison with the images obtained with the AM and EM algorithms. The FBP has been
included as it is a very commonly used algorithm for image reconstruction. This work has been published else were in grater details with experimental data (Varma et al. 2008).

### 3.1 The Algorithms

Since the objective of this study is to primarily compare the AM and the EM algorithms, these algorithms have been briefly discussed and outlined for the purpose of continuity. Also, as the FBP method is well established it has not been discussed here.

#### 3.1.1 Expectation Maximization Algorithm

Lange and Carson (Lange and Carson 1984) defined the image reconstruction for tomography as a maximum likelihood estimation problem and derived an estimation-maximization (EM) algorithm to obtain the maximum likelihood image estimate. In experiments where γ ray counts statistics is high (~ 500 counts/projection), ignoring the true statistical nature of the data may not be a serious limitation because Poisson counting noise is only a component of the total system noise (Lange and Carson 1984). It is precisely in the low counts experiments that the EM algorithm is expected to provide the greatest improvement in the reconstruction quality. Superiority of the EM algorithms over Fourier techniques and on iterative algebraic methods such as the incorporation of nonnegativity constraints and objective measure of quality of reconstruction (e.g. log-likelihood, least squares, maximum entropy) is discussed in detail by (Lange and Carson 1984).

The EM algorithm is a general iterative technique for computing maximum likelihood estimates in any general scenario of measurement of statistical quantities.
Application to image reconstruction in transmission tomography is only a specific application. Each iteration of the EM algorithm consists of two steps expectation (E step) and maximization (M step). The derivation of the E and M steps for transmission tomography are discussed in detail by Lange and Carson (Lange and Carson 1984). Some of the key elements are highlighted here to enable comparison with the AM.

Figure 3-1: Schematic of the representation of a transmission tomography domain, the boxes represent the pixels. Here S: source, D: detector, \( y \): projection (line measurement across the domain) index, \( x \): pixel index and \( h(y|x) \) is the segment of projection \( y \) in pixel \( x \) (Varma et al. 2008).

In the expectation step (E-step), conditional expectation of ‘entire’ or ‘complete’ data set is estimated on the basis of the measured data (counts from the detector) and the parameter set (attenuation values). The ‘complete’ data set represents intensity of the photons that enter and leave each pixel in the domain for all the projections. If for a given pixel \( x \) along a projection or line measurement \( y \) across the domain (Figure 3-1), we define \( X(y|x) \) as the random number of photons (or counts) that enter \( x \) and \( X(y|(x+1)) \) the random number that survive and leave \( x \), then this process can be modeled as a binomial distribution with two out comes: survival of the photon \( \exp(-b(y|x)\mu(x)) \) and absorption
of the photons \([1 - \exp(-\beta(y|x)\mu(x))])\]. Where \(\beta(y|x)\) is the segment of projection \(y\) in pixel \(x\). The probability of this binomial process can be represented by (3-1), in other words (3-1) represents the probability of the ‘complete’ data set for pixel \(x\).

\[
p_{x(x+1)} = \left( \begin{array}{c}
X(y|x) \\
X(y|x+1)
\end{array} \right) \left[ \exp(-\beta(y|x)\mu(x)) \right]^{X(y|x)} \left[ 1 - \exp(-\beta(y|x)\mu(x)) \right]^{X(y|x+1) - X(y|x)} 
\]

(3-1)

The first pixel along any projection \(y\) receives the photons emitted from the \(\gamma\) source. This is a Poisson process with mean \(I_s\) which represents the intensity of the source, its probability is given by

\[
p_{x(|y|)} = \frac{I_s^{X(y|\mu)} \exp(-I_s)}{X(y|\mu)}.
\]

(3-2)

Since the pixels are independent, the likelihood function of the entire ‘complete’ data (for all pixels along projection \(y\)) set can be represented as the product of the individual likelihood functions of each pixel that lies along a projection \(y\). This likelihood function is represented as (3-3).

\[
f(N|\mu) = I_s^{X(y|\mu)} \exp(-I_s) \prod_{x \in X} \left( \begin{array}{c}
X(y|x) \\
X(y|x+1)
\end{array} \right) \left[ \exp(-\beta(y|x)\mu(x)) \right]^{X(y|x)} \left[ 1 - \exp(-\beta(y|x)\mu(x)) \right]^{X(y|x+1) - X(y|x)} 
\]

(3-3)

If the complete data set was known then computing the attenuation values \(\mu(x)\) for every pixel would be straight forward. However, this is not the case since only the counts data observed by the detectors representing the random number of photons that have survived all the pixels along a projection is available. The expectation step involves the computing the expectation of the ‘complete’ data set for a given pixel, conditional to the
observed data \( d(y) \) (measured by detector D) given the current estimate of the attenuation function \( \hat{\mu}^{(k)}(x) \) as given by (3-4).

\[
E\left[X(y|x)d(y), \hat{\mu}^{(k)}(x) \right] \tag{3-4}
\]

Lange and Carson (Lange and Carson 1984) show that (3-4) can be represented as

\[
E\left[X(y|x)d(y)\right] = d(y) + E\left[X(y|x)\right] - E\left[d(y)\right]. \tag{3-5}
\]

Equation (3-5) is used to compute \( M(y|x) \) and \( N(y|x) \), the expected values of the photons entering and leaving given pixel \( x \) respectively along projection \( y \). The entities \( M(y|x) \) and \( N(y|x) \) are the conditional expatiations related to \( X(y|x) \) and \( X(y|x+1) \) respectively by (3-4).

In the M-step, this conditional expectation is maximized with respect to a parameter set. In the case of \( \gamma \) ray transmission tomography, the parameter set is the attenuation values \( \mu(x) \) of the pixels in the scanned domain and the measured data set \( d(y) \) is the detector signal obtained for each projection \( y \). Equation (3-5) is used to determine the conditional expectation values \( M(y|x) \) and \( N(y|x) \) for each pixel in the image which is then summed over all projection and then substituted in the likelihood function. The natural logarithm of this equation is given by (3-6) which represents the log likelihood of the ‘complete’ data set over all projections \( y \).

\[
\ln\left[ f(N(y): \mu) \right] = \sum_{y \in Y} \sum_{x \in X} \left\{ N(y|x) \ln[\exp(-b(y|x)\mu(x))] + (M(y|x) - N(y|x)) \ln[1 - \exp(-b(y|x)\mu(x))] \right\} + R \tag{3-6}
\]
$R$ represents all the terms that are not dependant on $\mu(x)$. Maximizing (3-6) by setting the value of the partial derivative with respect to $\mu(x)$ to zero yields a transcendental equation:

$$
\sum_{j \in Y} - N(y|x) \beta(y|x) + \sum_{j \in Y} [M(y|x) - N(y|x)] \frac{\beta(y|x)}{\exp[\beta(y|x) \mu(x)] - 1} = 0. \tag{3-7}
$$

Since (3-7) cannot be solved exactly due to the exponential term, Lange and Carson (Lange and Carson 1984) suggest the Taylor series based approximation to simplify the solution:

$$
\frac{1}{\exp[\beta(y|x) \mu(x)] - 1} = \frac{1}{\beta(y|x) \mu(x)} - \frac{1}{2} + \frac{\beta(y|x) \mu(x)}{12} + O\left\{[\beta(y|x) \mu(x)]^3\right\}. \tag{3-8}
$$

The authors recommend using the first three terms of (3-8) to arrive at (3-9) in order to make a good approximation of (3-7):

$$
0 = \mu(x)^2 \cdot \sum_{j \in Y} [M(y|x) - N(y|x)] \frac{\beta(y|x)^2}{12} - \mu(x) \cdot \sum_{j \in Y} [M(y|x) + N(y|x)] \frac{\beta(y|x)}{2} + \sum_{j \in Y} [M(y|x) - N(y|x)] \tag{3-9}
$$

Equation (3-9) is a quadratic equation which has a solution given by (3-10)

$$
\hat{\mu}(x)^{(k+1)} = B/2A - \sqrt{B^2 - 4AC / 2A} \tag{3-10a}
$$

Where, $A = \sum_{j \in Y} [M(y|x) - N(y|x)] \frac{\beta(y|x)^2}{12}$, $B = \sum_{j \in Y} [M(y|x) - N(y|x)] \frac{\beta(y|x)}{2}$, and

$$
C = \sum_{j \in Y} [M(y|x) - N(y|x)]. \tag{3-10b}$$
The iterative process starts with an initial guess for $\hat{\mu}(x)^k (k = 0)$, this is used to compute the conditional expectation values of $M(y|x)$ and $N(y|x)$ using $d(y)$ based on (3-5). These values are then used to determine the next estimate of attenuation $\hat{\mu}(x)^{k+1}$ based on (3-10). By setting $k = k + 1$ this process is repeated starting from (3-5) till the value of $\hat{\mu}(x)^{k+1}$ which maximizes the log likelihood of the complete data set is reached.

The approximation for (3-7) given by (3-8) as discussed in the previous paragraph, is valid for small values of $b\left(y|x, \mu(x)\right)$ and is not accurate for application involving regions of high density, high mass attenuation $\mu(x)/\rho$ (cm$^2$/g), or for coarse pixel resolution where the values of $b\left(y|x\right)$ are large. This in turn may lead to a decrease in the log-likelihood function from one iteration to the other instead of an increase. This is a major short coming for imaging applications involving multiphase flow systems as they are usually constructed with metals or have internal structures that are also made of metal. To improve accuracy other authors have used modified approaches to derive the M-step (Ollinger 1994) or the E-step (Browne and Holmes 1992) of the EM algorithm in the context of its application to positron emission tomography (PET). Although these approaches can be adapted to image reconstruction in $\gamma$ ray transmission tomography for further improvement, only the work by Lange and Carson (Lange and Carson 1984) has been compared here to the AM algorithm as there is a precedence of its use for determining phase holdup distribution in multiphase systems using $\gamma$ ray tomography (Al-Dahhan et al. 2007; Karim et al. 2004; Kemoun et al. 2001; Khopkar et al. 2005; Kumar 1994; Kumar and Duduković 1997; Kumar et al. 1997; Luo 2005a; Patel et al. 2007; Rados et al. 2005a; Roy 2006a; Roy and Al-Dahhan 2005).
3.1.2 Alternating minimization Algorithm

Expectation – maximization algorithm is a special case of Alternating – minimization (AM) (O'Sullivan and Benac 2007) algorithm. AM algorithm has been studied extensively in the applications involving x-ray based medical imaging (Benac 2005). O'Sullivan and Benac (O'Sullivan and Benac 2007) reformulated the maximum likelihood problem as a double minimization of an I-divergence to obtain a family of image reconstruction algorithms. I-divergence, introduced by Csiszár (Csiszár 1991), is a measure of discrepancy between two functions \( a(y) \) and \( b(y) \), which is given as:

\[
I(a\|b) = \sum_{y \in Y} \left[ a(y) \ln \left( \frac{a(y)}{b(y)} \right) - [a(y) - b(y)] \right] \tag{3-11}
\]

where \( y \) is a finite dimensional space.

Csiszár (Csiszár 1991) examined a wide variety of discrepancy measures (e.g. least squares, entropy) including the I-divergence measure (3-11) between two functions \( a(y) \) and \( b(y) \) and arrived at the following conclusion which is relevant for the image reconstruction problem subject to nonnegativity constraints. Csiszár (Csiszár 1991) concluded that if the functions involved are all real valued, having both positive and negative values, then minimizing the least squares is the only consistent choice; whereas, if all the functions are required to be nonnegative, then minimizing the I-divergence is the only consistent choice. This I-divergence measure was first employed for image reconstruction by Snyder et al. (Snyder et al. 1992).
Each step of minimization in the AM algorithm is claimed to be exact process, without any approximation as in the case of EM (3-8), which represents one of its advantages over the EM algorithm. The alternating minimization algorithm formulated by O’Sullivan and Benac (O’Sullivan and Benac 2007) is guaranteed to monotonically increase the log-likelihood function at every iteration. This image reconstruction algorithm is based on a statistical model for the measured data, Beer’s law and a realistic model for the known point spread function.

A brief outline of the algorithm is described below. More details and mathematical proof are available elsewhere (O’Sullivan and Benac 2007). If we define \( q(y: \mu) \) as the model for the transmission of \( \gamma \) photons per Beer Lambert’s law, then

\[
q(y: \mu) = I_s(y) \exp \left[ - \sum_{x \in X} b(y|x) \mu(x) \right].
\] (3-12)

Where \( I_s(y) \) represents incident photon counts (or intensity), \( b(y|x) \) is the point spread function, or the length of projection \( y \) in pixel \( x \). In the expression for the I-divergence, the function \( a(y) \) is taken to be the measured data represented by \( d(y) \) (a Poisson random number) and \( b(y) \) is taken to be a nonlinear model \( q(y: \mu) \) representing the transmission of the photons. The term \( q(y: \mu) \) is a function of the attenuation set \( \mu \), where \( x \) is the pixel space index.

If (3-11) can be written for this case as

\[
I(d||q(y: \mu)) = \sum_{y \in Y} \left\{ d(y) \ln \left[ \frac{d(y)}{q(y: \mu)} \right] - [d(y) - q(y: \mu)] \right\}.
\] (3-13)
The overall objective of the algorithm could be stated as:

$$\min_{\mu \in \mu} I(q\|q),$$

(3-14)

finding the minimum of $I(q\|q)$ with respect to $\mu$.

The terms in the log likelihood function that depend on the parameter set (attenuation values to be estimated) are negative of the corresponding terms in the I-divergence. Thus, minimizing the I-divergence over the parameter set $\mu$ is equivalent to maximizing the log-likelihood function. Minimizing the I-divergence offers the unique advantage as it has a known lower bound (equal to zero), for projections data processed from any system. This is not the case with maximizing the log likelihood as the upper bound is not known.

Minimizing (3-13) as per (3-14) yields (3-15), the iterative expression for updating the parameter set (attenuation values):

$$\mu^{(k+1)}(x) = \mu^{(k)}(x) - \frac{1}{Z(x)} \ln \left( \frac{\tilde{b}(x)}{\hat{b}^{(k)}(x)} \right).$$

(3-15)

The terms with $^\wedge$ indicate that it is an estimate of the entity. The terms $\tilde{b}$ and $\hat{b}^{(k)}$ are the back projections of $d$ and the current estimates of $q^{(k)}$ respectively. In other words, they are the back projections of the measured data and the nonlinear model employed (based on Beer-lambert's law). Their expressions are given by:

$$\tilde{b}(x) = \sum_{y \in Y} b(y|x) d(y),$$

(3-16)
\[ \hat{b}^{(k)}(x) = \sum_{y \in Y} b(y|x) \hat{q}(y), \quad (3-17) \]

where,

\[ \hat{q}^{(k)}(y) = I_y(y) \exp \left[ - \sum_{x \in X} b(y|x) \hat{\mu}^{(k)}(x) \right]. \quad (3-18) \]

The iterative process for computing the image goes backwards starting from (3-18) to (3-15). An initial guess for \( \mu^{(k=0)}(x) \) is chosen to calculate \( \hat{q}(x) \) in (3-18), then \( \hat{q}(x) \) is used to calculate \( \hat{b}(x) \) in (3-17). The back projection \( \tilde{b}(x) \) is computed just once based on the measured data \( \tilde{d}(y) \). \( \mu^{(k+1)}(x) \) is updated based on (3-15), and the process is started again setting \( k = k + 1 \) using the updated values of attenuation. A non-negativity constraint is applied on the valued of attenuation. Hence at any iteration, if \( \hat{\mu}^{(k+1)}(x) < 0 \) it is over written as \( \hat{\mu}^{(k+1)}(x) = 0 \). \( Z(x) \) in (3-15) is an appropriate scaling function chosen for the \( x \)-th pixel such that the following criteria is satisfied (O'Sullivan and Benac 2007):

\[ \sum_{x \in X} \left( \frac{b(y|x)}{Z(x)} \right) \leq 1. \quad (3-19) \]

For every pixel \( x \) the length of the longest projection \( y \) passing through it was chosen as the value of \( Z(x) \) such that (3-19) was satisfied. The values of \( Z(x) \) was found to be insensitive to the final convergence values.

The AM algorithm discussed here doesn’t model for Compton scatter. In this study \( \gamma \) ray data used includes the only unattenuated \( \gamma \) ray photons. If the scatter is significant and
unavoidable in the scanner then a more sophisticated algorithm must be used. If the mean scatter is known for a given case then mathematical strategies have been discussed by O'Sullivan and Benac (O'Sullivan and Benac 2007).

### 3.2 Data generation methodology

Transmission data from three different types of phantoms were processed by the algorithms for comparison. The first two cases of the phantom considered were synthetic, as in simulated $\gamma$ ray transmission data was generated on the basis of these phantoms and processed. In the third case experimental transmission data was gathered with a second generation fan beam tomography setup and processed. The holdup distribution images of the liquid and the gaseous phases in the three cases of the phantom are determined and the results are compared.

The first phantom has regions of gas (air) and liquid (water) alone. The second phantom has regions with solid (metal) in addition to gas (air) and liquid (water). The objective of using the second phantom is to test for applications where the solid phase in the system is stationary, like in case of $\gamma$ ray tomography applied to columns with unavoidable internal structures, or those with structured packing or fixed bed (Roy and Al-Dahhan 2005). In such applications the background scans, or calibration scans, of the column include the solid phase, therefore only the liquid and gaseous phases are accounted for when the dynamic system is imaged. In the third case experimental data was collected for a phantom with the gaseous phase (air) and liquid phase (air). Although the gas and liquid phase are stationary in all the phantoms, they are processed assuming them to be dynamic. Hence, the holdup images obtained are meant to provide a time averaged holdup distribution information.
3.2.1 Details of the phantom

In this study a synthetic phantom was considered with dimensions as indicated in Figure 2. The diameter of the phantom, \( D_{\text{col}} \) was set to 8 in., the other region in the phantom have dimensions that are in proportion to the diameter. The proportionality ratios are indicated in Figure 2. In case I, the regions R1 though R4 are filled with air and the background region B is filled with water (table 1). This represents an ideal two phase arrangement consisting of liquid and gas. In case II, the regions R1 and R4 are filled with air, regions R2 and R3 are filled with iron, and the background region is filled with water (Table 3-1).

![Figure 3-2: Schematic of phantom for which the data was generated by simulation. \( D_{\text{col}}=8 \) in. The details of the material in regions R1, R2, R3, and R4 are given in table 1 (Varma et al. 2008).](image)

The shape of the background region is the same as Case I. Case II represents situation where columns have internal structures that are static and are made of metal which
usually have high attenuation values. The holdup images are determined only for the gas and liquid present in the case II phantom.

Table 3-1: Materials used in the phantom and the attenuation values (Hubbell and Seltzer 1996).

<table>
<thead>
<tr>
<th>Region in Phantom (Figure 2)</th>
<th>Material used. Attenuation values indicated in parenthesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>Case II</td>
</tr>
<tr>
<td>B Water (0.0863 cm⁻¹)</td>
<td>Water (0.0863 cm⁻¹)</td>
</tr>
<tr>
<td>R1 Air (0.0 cm⁻¹)</td>
<td>Air (0.0 cm⁻¹)</td>
</tr>
<tr>
<td>R2 Air (0.0 cm⁻¹)</td>
<td>Iron (0.2197 cm⁻¹)</td>
</tr>
<tr>
<td>R3 Air (0.0 cm⁻¹)</td>
<td>Iron (0.2197 cm⁻¹)</td>
</tr>
<tr>
<td>R4 Air (0.0 cm⁻¹)</td>
<td>Air (0.0 cm⁻¹)</td>
</tr>
</tbody>
</table>

3.2.2 Details of the γ ray computer tomography (CT) scanner

Figure 3-3 shows the schematic of the γ ray CT considered in this study for simulating the counts transmission data for the phantom cases I and II (Figure 3-2) reported in this chapter. The dimensions are similar to a single fan with 9 detectors of the DE-DSCT setup discussed in Chapter 5. The scanner consists of a point ¹³⁷Cs γ ray source that has a photo peak at 661 keV. The attenuation data shown in Table 3-1 is based on 661 keV γ ray photons from ¹³⁷Cs.
Figure 3-3: Schematic of the scanner arrangement with phantom in the center considered for the study reported in this chapter. The region occupied by the detectors is indicated as detector array. The source, and the detector array, is moved along the locus as indicated for 197 positions and projection data for each location is gathered (Varma et al. 2008). Further details available in Chapter 5.

An equiangular fan beam arrangement of the source and the detectors is used. The source is at the pointed end of the fan and the detectors are at the curved end of the fan placed 120 cm apart. This detector has 9 detectors that moves long the arc of the detector array 21 times to create 189 detectors positions effectively for each source position. The projection is modeled as fine line between the source and the detector as the open area of the detectors is very small. A total of 197 source positions (also called views) are considered, hence γ ray counts data for 189 x 197 (total 37233) projections passing through domain are simulated. Further details of the scanner are available in Chapter 5. An 80x80 pixels resolution is used to reconstruct the image. Hence each pixel represents an area of 2.54 mm x 2.54 mm of the phantom.
3.2.3 \( \gamma \) ray counts data simulation

\( \gamma \) ray counts data was generated by simulation for Case I and II of the phantom. Schaffler’s (Schaffer 1970) algorithm is used to generate the counts \( I \) that are Poisson random numbers with probability \( P_I \) given by (3-20), and mean \( g(y: \mu_{\text{phantom}}) \) given by (3-21). This number can also be generated using the `poissrnd` from Matlab\textsuperscript{®} with (3-21) as the mean.

\[
P_I = \frac{\left[ g(y: \mu_{\text{phantom}}) \right]^I}{I!} \exp\left[ -g(y: \mu_{\text{phantom}}) \right] \quad (3-20)
\]

\[
g(y: \mu_{\text{phantom}}) = I_0(y) \exp\left[ -\sum_{x \in X} b(x|y) \mu_{\text{phantom}}(x) \right] \quad (3-21)
\]

This Poisson number generated for each \( y \) is set equal to \( d(y) \), the counts received by the detector. The attenuation value \( \mu_{\text{phantom}}(x) \) used to generate the counts data is the attenuation of the \( x^{th} \) pixel in the phantom (Figure 3-1) along projection \( y \). The algorithm (AM and EM) determines these values without any prior information about the phantom and based purely on the Poisson numbers.

To reduce the effect of noise and uncertainty in the data and to get a better quality of the images, the mean value of the counts based on multiple samples or readings for a given projection \( y \) is often used for processing the data, as represented by (3-22).

Here \( n \) represents the number of data sample of counts for a given projection \( y \),

\[
d(y) = \frac{1}{n} \sum_{i=1}^{n} I_i . \quad (3-22)
\]
If an infinite number of samples are collected then (3-23) would give an accurate value of the mean $q(y: \mu_{\text{phantom}})$,

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} d(y) = q(y: \mu_{\text{phantom}}). \quad (3-23)$$

Hence when (3-22) is used with some small finite values of $n$ then it can be said that an approximate estimate of the mean $q(y: \mu_{\text{phantom}})$ is used to process the data. To see how the number of data sample used to arrive at the mean value of the counts effects the AM and EM algorithms performance, the mean of multiple samples ($n$) of counts (ranging from 1 to 100) are used as estimates of $q(y: \mu_{\text{phantom}})$ for $d(y)$ as per (3-22). It can be seen as: higher the number of samples lesser the noise in the data. The maximum values of $n=100$ is an optimum choice as the results discussed in section 3.4.2 show that the maximum difference in reduction of noise, and by extension error, in the holdup images occurs for values lower values of $n$. Hence the reduction in error from $n=1$ to $n=5$ is far greater than the reduction in error from $n=50$ to $n=100$. The value for $n=100$ is also practical when data is collected with the scanner, as higher values would dramatically increase the time required to execute the scan without an increase in accuracy that is commensurate with it.

### 3.3 Calculation of holdup distribution

The counts data generated by simulation is processed by the algorithms (AM and EM) which reconstruct an image that depicts attenuation of the gas-liquid phantom, represented by $\mu_{\text{g-\ell}}(\infty)$. This attenuation value is a linear sum of the product of the holdup fractions of the phases and their pure attenuation values as given by (3-24).
\[
\hat{\mu}_{l-g}(x) = \hat{\mu}_l(x) \varepsilon_l(x) + \hat{\mu}_g(x) \varepsilon_g(x).
\] (3-24)

The subscripts \(l\) and \(g\) represent liquid and gas respectively, \(\varepsilon(x)\) represents the holdup fraction and \(\hat{\mu}(x)\) the attenuation of the pure phase. The objective of using CT is to determine the values of \(\varepsilon(x)\). The \(\hat{\mu}(x)\) values for pure liquid and gas are usually determined by a background scan where the domain purely consists of only one of the phases. Hence \(\hat{\mu}(x)\) for gas and liquid has been indexed by \(x\) on the right hand side of (3-24). If the attenuation values are known from literature or databases then they can be directly used. In that case the equation would be represented as:

\[
\hat{\mu}_{l-g}(x) = \mu_l \varepsilon_l(x) + \mu_g \varepsilon_g(x),
\] (3-25)

where \(\mu\) is not indexed with \(x\) as it doesn’t represent an image. However in this study only (3-24) is considered. In addition it is also known that the sum of the holdup fraction of the two phases add up to unity (3-26).

\[
\varepsilon_l(x) + \varepsilon_g(x) = 1.
\] (3-26)

To determine the gas holdup for the case I phantom, equations (3-24) and (3-26) are combined with the assumption that \(\hat{\mu}_g(x)\) (representing calibration attenuation image of the system filled with gas) to be equal to zero, to give (3-27). This assumption is reasonable as air has negligible contribution to the attenuation of a \(\gamma\) ray photon with 661 keV energy. Hence gas holdup distribution image is determined using (3-27), and by extension the liquid holdup distribution image by using (3-28)
\[ \varepsilon_i(x) = \frac{\mu_i(x) - \tilde{\mu}_{i-g}(x)}{\tilde{\mu}_i(x)} \] (3-27)

\[ \varepsilon_i(x) = 1 - \frac{\mu_i(x) - \tilde{\mu}_{i-g}(x)}{\tilde{\mu}_i(x)} \] (3-28)

In (3-27) and (3-28), \( \tilde{\mu}(x)_{i-g} \) represents the attenuation image of the phantom with region B filled with water and regions R1 through R4 (Figure 3-2) filled with air, and \( \mu(x)_i \) represents the attenuation image of the phantom with region B and R1 through R4 filled with water.

For the case II of phantom which has solid internals, attenuation images \( \tilde{\mu}(x)_{i-g} \) and \( \mu(x)_i \) in (3-27) are replaced with \( \tilde{\mu}(x)_{i-g-s} \) and \( \mu(x)_{i-s} \), respectively to give (3-29).

\[ \varepsilon_i(x) = \frac{\mu_{i-s}(x) - \tilde{\mu}_{i-g-s}(x)}{\tilde{\mu}_{i-s}(x)} \] (3-29)

The image \( \tilde{\mu}(x)_{i-g-s} \) has the liquid in region B, the gas in regions R1 and R4, and the solid in regions R2 and R3 of the phantom (Figure 3-2). The background scan image \( \mu(x)_{i-s} \) has the liquid in regions B, R1 and R4, and solid in regions R2 and R3 of the case II phantom (Figure 3-2)

Similarly, to determine the liquid holdup image for case II of the phantom, (3-28) is modified substituting \( \tilde{\mu}(x)_i \) with \( \left[ \tilde{\mu}_{i-s}(x) - \tilde{\mu}_{i-g-s}(x) \right] \) to give (3-31). The image \( \mu(x)_{i-s} \) has gas in regions B, R1 and R4, and solid in regions R2 and R3 of the case II phantom (Figure 3-2).
This way the solids is eliminated from the case II phantom as it is supposed to represent the background, or a constituent of the internals of the system. The region occupied by the solids in Figure 3-2 should now appear as part of the gas or the liquid holdup images.

The attenuation image $\mu(x)$ for different configurations as indicated above are calculated using the AM and EM algorithms. The holdup distribution images are determined post attenuation image reconstruction based on (3-27) through (3-30), for the liquid and the gaseous phases for two cases of the phantom. Although the algorithm reconstructs the attenuation images, the holdup distribution images obtained on its basis are analyzed to evaluate the AM and EM algorithms. This is done as ultimately the phase holdup images are of interest to obtain quantitative information about the system being scanned.

### 3.4 Results and discussion

The holdup images reconstructed based on simulated transmission data for case I and case II, are analyzed in the section. The mean percentage error of all the pixels in a given domain with a phantom are used as parameters to qualitatively assess the images. The error values used in this discussion is calculated by comparing the ideal values of holdup with the values of holdup from the reconstructed images. Further analysis in terms of standard deviation have been reported else where (Varma et al. 2008).
3.4.1 Gas holdup images

Figure 3-4: Gas holdup images obtained with AM algorithm (after 1000 iterations with $n=50$). The color bar represents the color code used for representing gas holdup values in the images. A: Gas holdup based on (3-27) for case I phantom (Figure 3-2), B: gas holdup based on (3-29) for case II of the phantom (Figure 3-2) (Varma et al. 2008).

Figure 3-5: Gas holdup images obtained with EM algorithm (after 1000 iterations with $n=50$). The color bar represents the color code used for representing liquid holdup values in the images. A: Gas holdup based on (3-27) for case I phantom (Figure 3-2), B: gas holdup based on (3-29) for case II of the phantom (Figure 3-2)
Using (3-27) and (3-29) the gas holdup images were successfully obtained. Figures 3.5, 3.6 and 3.7 show the gas holdup images obtained using AM, EM and FBP algorithms respectively.

Clearly both the AM and EM algorithms are able to successfully image the gas phase distribution in the two cases of the phantoms as the gas filled regions are captured in the image with a holdup values close to unity. There is no distortion in the reconstructed images, the geometric shape of the domains (R1 through R4) for case I, and R1 and R2 for case II are maintained and appear just as in the phantom. The transition in the gas holdup values at the edge of the domain is sharper in images generated by AM algorithm (Figure 3-4) as compared to those by the EM algorithm (Figure 3-5). The AM algorithm provides gas holdup mages with uniformity in the domains of the phantom (for both cases of the phantom) where the gas in present. These images also confirm that (3-29) is correctly formulated as complicated arrangements that have parts of the domain covered by the solid material (Figure 3-5B) in the phantoms are not misrepresented as part of the gas holdup.

Figure 3-6: Gas holdup images obtained using FBP (with n=50). The color bar represents the color code used for representing liquid holdup values in the images. A: Gas holdup based on (3-27) for case I phantom (Figure 3-2), B: gas holdup based on (3-29) for case II of the phantom (Figure 3-2) (Varma et al. 2008).
In contrast the FBP results (Figure 3-6) show artifacts. The circular domains R1 and R4 in cases I and II create a shadow in the image. Clearly the AM and EM gas holdup images have lesser error than the FBP image.

3.4.2 Quantitative analysis of gas holdup

A quantitative analysis of the images is possible only when the holdup values of all pixels in a given part of the image is analyzed for its error and standard deviation. Standard deviation indicates the level of noise in the image. If the standard deviation of the pixels is high, it means the image is noisy and a lot of freckles or grains can be seen. The most desirable situation is the one where both the standard deviation and the error are nil. When the standard deviation is small and the error is high then there is a bias in the image. The detailed of the images with the standard deviation have been reported elsewhere (Varma et al. 2008).

Figure 3-7 shows the mean percentage error for the two cases of the phantom obtained using both, the AM and the EM algorithms. Clearly the error values in Figure 3-7 shows that the AM- algorithm performs better than the EM algorithm for any condition of, the number of data samples or the iterations covered in both the phantoms. When transmission data with low levels of noise (n=100 in (3-22)) is used the EM algorithm still gives higher error than the AM algorithm. For case II of the phantom (Figure 3-7B) the difference in error between the two algorithms is almost an order of magnitude. This indicates that in the presence of high attenuation material the AM algorithm gives a more accurate estimate of gas holdup.
3.4.3 Liquid Hold up

The liquid holdup images were successfully obtained by using (3-28) for case I of the phantom and (3-30) for case II of the phantom. Figure 3-8, Figure 3-9, and Figure 3-10, shows the liquid holdup images obtained using AM, EM and FBP algorithms respectively. Clearly the liquid phase is separated successfully in these images. The results from the AM and EM algorithms show that there is no distortion in the images, the shapes of the region with liquid in the phantoms is successfully captured. The regions of the case II phantom that have the solids are not misrepresented as liquid holdup in images. As in the case of the gas holdup images the liquid holdup images have sharper transitions at the edge of the domain when AM algorithm is used.
Figure 3-8: Liquid holdup images obtained with AM algorithm (after 1000 iterations with $n=50$). The color bar represents the color code used for representing liquid holdup values in the images: A: Liquid holdup based on (3-28) for case I phantom (Figure 3-2), B: liquid holdup based on (3-30) for case II of the phantom (Figure 3-2) (Varma et al. 2008).

Figure 3-9: Liquid holdup images obtained with EM algorithm (after 1000 iterations with $n=50$). The color bar represents the color code used for representing liquid holdup values in the images: A: Liquid holdup based on (3-28) for case I phantom (Figure 3-2), B: liquid holdup based on (3-30) for case II of the phantom (Varma et al. 2008).
Figure 3-10: Liquid holdup images obtained using FBP method (with n=50). The color bar represents the color code used for representing liquid holdup values in the images: A: Liquid holdup based on (3-28) for case I phantom (Figure 2), B: liquid holdup based on (3-30) for case II of the phantom (Varma et al. 2008).

For the case II phantom, the liquid holdup image obtained with the EM algorithm (Figure 3-9B) has freckles, hence has more noise, as compared to the one obtained with AM algorithm in Figure 3-8B. Also the holdup in the region occupied by the high attenuation material (R3 and R4 in Figure 3) in the EM algorithm image is slightly above zero. Both these aspects are attributed to the presence of the high attenuation material present in the domain.

The FBP holdup images show artifacts similar to the gas holdup images. There is a shadow created by the circular gas holdup regions of the domain. For the case II phantom the metallic region R2 and R3 created the most pronounced artifacts. The circular shape of domains R3 and R4 are lost and most of this region appears as part of the liquid holdup distribution (Figure 3-10B). The high attenuation domain severely affects the FBP results.
3.4.4 Quantitative analysis of liquid holdup

Figure 14 shows the mean percentage error in estimating the liquid holdup values in region of the phantoms filled with the liquid. Clearly for all three cases of the phantom the AM algorithm performs better universally than the EM algorithm. For the case II of the phantom there is an order of magnitude difference in the error between the holdups values results from both the algorithms.

Figure 3-11: Mean percentage error in liquid holdup for results obtained with AM and EM algorithms for projection data with n=1 through n =100. A: Error in liquid holdup for pixels of region B (Figure 3-2) of the case I phantom, B: error in liquid holdup value for region B for case II (Figure 3-2) of the phantom and (Varma et al. 2008).

Also, the error doesn’t stabilize or reduce for data with higher noise levels (lower values of n as per (3-23)) when the EM algorithm is used for this case of the phantom. It should be noted that the liquid domain in the phantoms of cases I and II are physically similar. This clearly shows that holdup image of a phase that doesn’t have a high attenuation is effected by parts of the domain that have the high attenuating material, or phase. The AM algorithm provides images with better quality in such applications as compared to the EM algorithm.
3.5 Conclusions

The AM algorithm was successfully implemented for the case of single energy $\gamma$ ray tomography to determine phase holdup images for two phase systems for phantoms with different configurations. The performance of the algorithm was compared by putting the effect of some critical parameters such as, the noise level in the data and the number of iterations used to reconstruct the image, in perspective. A systematic quantitative analysis of the holdup distribution images generated using the AM and EM algorithm were carried out for the three cases of the phantom. The AM and EM images were compared to the FBP images. The AM algorithm proposed by O'Sullivan and Benac (O'Sullivan and Benac 2007) performs better than the EM algorithm proposed by Lange and Carson (Lange and Carson 1984) when used for $\gamma$ ray tomography to determine holdup images in two phase systems. The holdup image obtained with FBP are less accurate than the AM and EM results. The results show an overall improvement in the quality of the image in terms of the noise and accuracy of the estimated values when the AM algorithm is used. For data with high noise the AM algorithm gives lesser error when compared to EM and FBP algorithms, and it requires less number of iterations to reach a given level of accuracy. Applications that involve the use of high attenuation material the AM algorithm is more stable and produces holdup images that have a greater degree of accuracy and lower levels of noise. This is true for both, parts of the domain that have the high attenuation material (metals) and parts that don’t have it, like the surrounding areas (liquid and gas).

The differences in the performance of the two stochastic algorithms could be attributed to the simplification introduced in the M-step of the EM algorithm as indicated in
(3-8) to estimate (3-7). In spite of using three terms of (3-8) (shown in (3-9)) for EM algorithm, as recommended by (Lange and Carson 1984) for maximum accuracy in the results, it doesn’t match the performance of the AM algorithm the AM algorithm performs better. Hence the AM algorithm is a better choice for image reconstruction for determining the holdup distribution images in multiphase systems involving two phase flow.
Chapter 4

Algorithm for Imaging Phase Holdup Distribution in Three Phase Flow

4.1 Introduction

Imaging reconstruction and the process for determining the phase holdup distribution for a two phase system was demonstrated in Chapter 2. The attenuation coefficient of the mixed phase (mixed flow image) are used to determine the phase holdup distribution based on the constitutive equations that relate phase holdup to the attenuation coefficient of the mixed flow. The expectation maximization (EM) algorithm was compare to alternating minimization (AM) algorithm for single energy $\gamma$ ray computed tomography (CT) application. The accuracy of the results depends on the accuracy of the attenuation images.

In this chapter the discussion is extended to imaging phase holdup distribution in three phase flow with the objective of determining an algorithm or method to determine the phase holdup distribution images. The algorithm studies have been carried out with simulated $\gamma$ ray transmission data from a synthetic three phase phantom. The simulation accounts for the noise in the transmission data due to the stochastic nature of the $\gamma$ photon transmission process. Other sources of noise actually introduced experimental transmission data due to the system hardware or other sources have not been included. A simulated environment is chosen for the algorithm studies as, like any simulation, complications due
external distortions that influence experimental data are avoided such. This way the fundamental characteristics of the algorithm can be better understood. Two appropriate $\gamma$ photons from the higher energy range (greater than 150 keV) are selected for the study. The general approach followed in literature, which is an extension of the approach followed for two phase systems, has been applied to this case.

4.2 $\gamma$ photon energy and isotope selection

One the important aspect of the development of a dual energy tomography system is the selection of appropriate $\gamma$ photons based on its energy. Some of the desirable properties would include: a high signal to noise ratio, sufficient contrast in the attenuation values of the phases of interest and the physical availability of an isotope that emits the $\gamma$ photons. Figure 4-1 shows the profile of the attenuation versus energy of the $\gamma$ photon for water and two types of glass, lead and pyrex (borosilicate) representative as a lower bound and upper bound of surrogates for catalyst particles typically used in cold flow studies. The plot has been generated based on the data from NIST (Hubbell and Seltzer 1996) database. The inverse of the attenuation would represent the mean penetration depth of the photons. If the penetration dept of a $\gamma$ is too low as the domain size get lager the signal to noise ratios get poorer. For the attenuation profiles or the materials discussed in Figure 4-1, it can be seen clearly that that at lower energies the attenuation values are exceptionally high.
Figure 4-1: The attenuation (in cm$^{-1}$) versus energy profile for water, lead glass and pyrex glass. The data was obtained from NIST (Hubbell and Seltzer 1996). The vertical lines indicate the energies of the $\gamma$ photons from the respective radio isotopes indicated.

The energy of the $\gamma$ photons from some of the radio isotopes available commercially have been indicated as vertical lines in Figure 4-1. Further details on the half life and availability of the radio isotopes have been discussed in Chapter 5. Clearly $\gamma$ photons from isotopes like $^{241}$Am and $^{75}$Se are attenuated to a large extent by the materials considered. If these sources are used then very high strengths would be required to conduct scans of large domains (~6 inches and above). The radiation hazards associated with it would be significant operational challenge. Of the available isotopes the $^{137}$Cs and $^{60}$Co have the $\gamma$ photons that would not be attenuation during transmission and be able to provide a reasonable amount of photon counts.

In this simulation study the 1332 keV energy photons from $^{60}$Co and the 661 keV photons from $^{137}$Cs, have been considered. The transmission data has been generated based
on the attenuation data of the materials in question for $\gamma$ photons of the above mentioned energies.

### 4.3 Details of the Three Phase Phantom Considered

In this section the details of synthetic phantom and the scanner configuration used to generate the simulated transmission data, are discussed. Figure 4-2 shows the schematic of the phantom truth image. The main domain – B is a circular region with a diameter of 8 inches and has four circular subdomains; C1 through C4. The respective dimensions and positions of the subdomains are indicted in Figure 4-2. The holdup values of the phases in the domains of the phantom are given in Table 4.1. As most multiphase systems are cylindrical in shape, the main domain of the phantom is made circular to resemble a 2D section it. Subdomains C1 and C3 are solids rich and have a similar composition and subdomains C2 and C4 are gas rich and have a similar composition.

Typically cold flow experimental multiphase studies use air and water as the liquid and gas phases respectively. Fine glass beads are used as surrogate catalyst particles which constitute the solid phase. Since catalyst particles typically have a wide range of metallic elements and densities and extension attenuation, two cases of the phantom were considered: case I consisting of air, water and borosilicate glass (pyrex glass), and case II consisting of air, water and lead glass. Borosilicate glass ($\rho = 2.3\text{kg m}^{-3}$) and lead glass ($\rho = 6.2\text{kg m}^{-3}$) are reasonable surrogates of catalyst particles representing a lower and upper bound in terms of attenuation. The attenuation data at the energies considered are available in Table 4.2. An 80 × 80 pixel resolution was used for image reconstruction. This
translates to a resolution of 2.54 mm for every pixel. The pixels in the circular domain B in Figure 4-2 alone were reconstructed.

Figure 4-2: Schematic of phantom truth: The phantom consists of a main circular domain B with circular subdomains C1 – C4. The phase holdup values for all the domains are given in Table I. The vertical line indicates where the profiles have been shown in Figs 3 and 5.

<table>
<thead>
<tr>
<th>Region in Phantom (Fig 1)</th>
<th>Gas holdup</th>
<th>Liquid holdup</th>
<th>Solid Holdup</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.3</td>
<td>0.6</td>
<td>0.1</td>
</tr>
<tr>
<td>C1 and C3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>C2 and C4</td>
<td>0.6</td>
<td>0.1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 4.2: Attenuation values of the materials used in the phantom for the gamma energies considered (Hubbell and Seltzer 1996).

<table>
<thead>
<tr>
<th>Energy</th>
<th>Air (cm$^3$)</th>
<th>Water (cm$^3$)</th>
<th>Borosilicate glass (cm$^3$)</th>
<th>Lead Glass (cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>661 ($^{137}$Cs)</td>
<td>0.000091</td>
<td>0.0863</td>
<td>0.1865</td>
<td>0.6494</td>
</tr>
<tr>
<td>1332 ($^{60}$Co)</td>
<td>0.000065</td>
<td>0.0638</td>
<td>0.1194</td>
<td>0.3667</td>
</tr>
</tbody>
</table>
The projection sinograms, shown in Figure 4-3, were simulated for a proposed dual energy γ ray CT discussed in Chapter 5, with a 2D fan beam arrangement of source and detectors. The sinogram represents the image in the projection space (or y space in terms of notation), the ordinate presents the source position of view and the abscissa represents the projection number. Each pixel in the image is the transmission ratio for the corresponding source position and projection. The procedure described in Chapter 3 section 3.2.3 for the generation of Poisson random numbers that represents the photons counts based in the phantom geometry and composition were followed.

Figure 4-3: Sinogram images of the phantom generated by simulation. A: Sinogram of 1132 keV photon from $^{60}$Co isotope. B: Sinogram of 661 keV photon from $^{137}$Cs.

A total of 197 source positions were considered with 189 projection, or line measurements across the domain, were used to generate the sinograms. Hence, the sinogram for each energy of the gamma photons considered has 37233 projections through the phantom. The incident photon counts for each energy $I_o(y, E)$, was assumed to be $2 \times 10^3$ s$^{-1}$, the approximate counts received by the detectors of the proposed scanned (discussed in Chapter 5) were used. The photons counts generated were assumed to represent the
unattenuated counts measured by the detectors, the scattered photon counts are not included in this study.

4.4 Monoenergetic Approach for Dual Energy $\gamma$ Ray Tomography (ME)

The phase holdup distribution for a two phase system is determined based on the attenuation image of the flow and the fact that the sum of the phase holdups is equal to unity. The attenuation coefficient of the pure phase also needs to be known. The phase holdup distribution for three phase flow is also determined along similar lines.

The attenuation coefficient of the dispersed flow containing a gas – liquid – solid can be presented as:

$$\mu(x, E) = \sum_{i \in \{G, L, S\}} \mu_i(E) \varepsilon_i(x) . \quad (4-1)$$

The terms $\mu(x, E)$ represents the attenuation coefficient image of the mixed flow, $\mu_i(E)$ represents the attenuation coefficient of the phase indexed by $i$, where $i \in \{G, L, S\}$. The terms $E$ and $x$ are the indices of energy of the photon and image space (pixels). Since phase holdup is a fraction of the volume of the given phase, the sum of the phase holdups fractions is unity, which is represented as:

$$\sum_{i \in \{G, L, S\}} \varepsilon_i(x) = 1 . \quad (4-2)$$

$\varepsilon_i(x)$ represents the phase holdup of the phase indexed by $i$ in the pixel $x$. 
If the flow domain in imaged by γ photons of two different energies indexed as $E = 1$ and $E = 2$, the two equation are available in the form of (4-1). These two equations along with (4-2) provide a closed system of equations to determine the phase holdup distribution of each of the phases present in the flow. This could be represented in the matrix form as \( \hat{\mu} = M \hat{s} \), or:

\[
\begin{bmatrix}
\hat{\mu}^{(k+1)}(x,1) \\
\hat{\mu}^{(k+1)}(x,2)
\end{bmatrix} =
\begin{bmatrix}
\mu_s(1) & \mu_L(1) & \mu_G(1) \\
\mu_s(2) & \mu_L(2) & \mu_G(2) \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{\varepsilon}_s(x) \\
\hat{\varepsilon}_L(x) \\
\hat{\varepsilon}_G(x)
\end{bmatrix}
\] (4-3)

where,

\[
M = \begin{bmatrix}
\mu_s(1) & \mu_L(1) & \mu_G(1) \\
\mu_s(2) & \mu_L(2) & \mu_G(2) \\
1 & 1 & 1
\end{bmatrix}.
\] (4-4)

If the terms in the matrix on the left of (4-3), and values of coefficients that constitute \( M \) are known, then the holdups can be determined by inverting \( M \). This could be presented as:

\[
\begin{bmatrix}
\hat{\varepsilon}_s(x) \\
\hat{\varepsilon}_L(x) \\
\hat{\varepsilon}_G(x)
\end{bmatrix} =
\begin{bmatrix}
\mu_s(1) & \mu_L(1) & \mu_G(1) \\
\mu_s(2) & \mu_L(2) & \mu_G(2) \\
1 & 1 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
\hat{\mu}^{(k+1)}(x,1) \\
\hat{\mu}^{(k+1)}(x,2)
\end{bmatrix}.
\] (4-5)

The iterative procedure described in section 3.1.2 based on (3-15) is used to determine the attenuation coefficient images \( \hat{\mu}^{(k+1)}(x,1) \) and \( \hat{\mu}^{(k+1)}(x,2) \) for energies $E = 1$ and $E = 2$ of the mixed flow. The values of the terms in \( M \) can be obtained from the literature if the composition of the phase is known exactly. If the data is unavailable then the domain is usually filled with a single phase and scanned. The transmission data could then be processed using (3-15) to determine the attenuation coefficient of the pure phase, where its holdup is unity. In the latter case \( M \) can be represented as:
Here $\mu_i(x, E)$ for $i \in \{G, L, S\}$ and $E \in \{1, 2\}$ represents the attenuation coefficient of the domain filled completely with the component $i$ when:

$$
\varepsilon_i(x) = 1.
$$

The terms that have a ^ in (4-5) represent and estimate of the variable. Since the holdup $\varepsilon_i(x)$ is calculated based on the estimate of attenuation coefficient of the mixed flow it is represented as the estimate too.

A suitable algorithm is used to determine the attenuation coefficient images of the flow: $\hat{\mu}^{(k+1)}(x, 1)$ and $\hat{\mu}^{(k+1)}(x, 2)$. The terms in $\mathbf{M}$ are determined based on the attenuation data from literature or calibration scans. Equation (4-5) is used to determine the holdup images.

Since image reconstruction of the attenuation coefficient image in the procedure described is an intermittent step to determine the phase holdup distribution images, this process is referred as a post processing method of determining phase holdup distribution. This post processing approach is called the monoenergetic (ME) approach to the dual energy problem in this discussion as the image reconstruction is done with data from a single energy of $\gamma$ photon at a time. The post processing based on the ME approach for determining the phase holdup distribution is the standard method reported in literature for dual energy $\gamma$ ray (Bukur et al. 1996; Froystein et al. 2005; Hale 2007; Nikitidis et al. 1999; Rebgetz et al. 1991; Rizescu et al. 2001; Yazdi and Esmailnia 2003) and x ray CT (Gehrke and Wirth 2005; Hu et al. 2005; Rapaport et al. 1995).
The algorithms used for determining the attenuation images $\mu(x, E)$ of the flow have differed in literature. Filter back projection (FBP) based algorithms have are the most commonly sighted. In this study the monoenergetic alternating minimization (ME-AM) as described in section 3.1.2 of Chapter 3 has been applied. The iterative procedure based on equation (3-5) was used to process the data represented by the sinograms in Figure 4-3 to determine the attenuation coefficients $\hat{\mu}^{(k+1)}(x,1)$ and $\hat{\mu}^{(k+1)}(x,2)$. A random number between 0 and 1 was chosen as used as the initial guess for $\hat{\mu}^{(k=0)}(x, E)$. A total of 15000 iterations were carried out to determine $\hat{\mu}^{(k+1)}(x,1)$ and $\hat{\mu}^{(k+1)}(x,2)$. The attenuation data in Table 4.2 was used for the respective terms of $\mathbf{M}$ in (4-6). Based on this, (4-5) was used to estimate the phase holdup distribution in the phantom (Figure 4-2).

### 4.5 Results of Monoenergetic Approach (ME-AM)

The holdup distribution images were obtained using (4-5) after $10^4$ iterations of (3-15) to determine the images $\hat{\mu}$. The current values of the I-divergence were calculated based on (3-13) using the current values of $q^{(k)}(y, E)$ from (3-18) at every iteration $k$. This was done for both the energies. The plots of I-divergence versus iterations are shown in Figure 4-4 for cases I and II of the phantom.
Clearly the figures indicate the minimization of the I-divergence as the iterations progress, thereby indicating convergence of the algorithms. The I-divergence reaches its minimum value in range of 1000 iterations. This case was run for $10^4$ iterations to ensure that an asymptotic value is reached. Hence the converged values of $\hat{\mu}$ were used in (4-5).

The holdup image results calculated based in (4-5) for every pixel in the image are shown in Figure 4-5. Since the maximum holdup of any phase in the phantom is 0.6 (Table 4.1) a color scale with a range of 0.0 to 0.75 was chosen. The first row of images in Figure 4-5 show the ideal values of gas, liquid and solid holdups respectively. The second and third rows in Figure 4-5 show the images obtained with lead glass (case I) and pyrex glass (case II) as the solid phase in the phantom respectively. The columns A B and C in Figure 4-5 represent the gas, liquid and the solid holdups respectively.
Figure 4-5: Holdup image reconstructed using ME approach. Case I phantom with lead glass as solid phase. Columns: A: gas holdup, B: liquid holdup and C: solid holdup.
The images are of poor quality and appear highly grainy. The holdup images of the case I phantom with has lead glass are slightly better than the case II phantom with pyrex glass. The profiles through the images (along the vertical line in Figure 4-2), are shown in Figure 4-6. This is done to assess the accuracy of the image as ideally the phase holdup distribution profile should match the truth. The scale of the plot has been forced between 0 and 1 as the holdup values are expected to be in that range. The profiles in results (Figure 4-6) indicate that the holdup values at a lot of instances along the section are actually negative or above unity. This discrepancy is more pronounced in case II of the phantom. These results are highly inaccurate and meaningless as holdup, being a fraction, has to have a value between 0 and 1.

Figure 4-6: Profile of ME-AM reconstructed images along the vertical line indicated in the phantom (Figure 4-2) showing the phase holdup values for gas, liquid and solid.
The phase holdup images in Figure 4-5 don’t provide qualitative information, especially in case II where the boundaries of subdomains C1 – C4 are not clearly visible. The profiles in Figure 4-6 indicate that the quantitative information on the holdup distribution in the phantom is also very poor as there is a lot of error.

The holdup results are obtained from a system of linear equations (4-5) which are independent. Hence there should be no error. However there is a lot of error in the phase holdup images $\hat{\epsilon}$. This is attributed to the ill conditioned nature of the matrix $M$ in (4-4) or (4-6). The ill conditionality could be characterized in terms of the condition number $\kappa$ defined as

$$\kappa(M) = \|M^{-1}\| \cdot \|M\|,$$  \hspace{1cm} (4-8)

where $\|\|$ is the spectral norm. For cases I and II of the phantom (with lead glass and pyrex glass as the solid phase respectively) the numerical value of $\kappa(M)$ is 181.6 and 19586.2, respectively. The attenuation constants of the materials used in the phantom for the gamma photons from the isotopes $^{60}$Co and $^{137}$Cs determine the values of $\kappa(M)$. A high $\kappa(M)$ indicates that $M$ ill conditioned.

The matrix $M$ causes small errors in the estimation of $\hat{\mu}^{(k+1)}(x,1)$ or $\hat{\mu}^{(k+1)}(x,2)$ computed from (3-15)) amplify when the holdup images $\hat{\epsilon}$ are determined using (4-5). The small errors in the estimate of $\hat{\mu}^{(k+1)}(x,1)$ or $\hat{\mu}^{(k+1)}(x,2)$ is inevitable due to the stochastic nature of the $\gamma$ photon emission processes or the randomness associated with it. The poorer quality of holdup images for case II of the phantom than case I is explained by the higher condition number of the $M$ associated with it. In Chapter 3 it was established that The ME images $\hat{\mu}$ computed with AM (using (3-15)) are more accurate than the ME images computed by EM and FBP algorithms (Varma et al. 2008), or nonimaging methods such as
gamma ray densitometry that directly invert the Beer-Lambert’s equation (Bukur et al. 1996). Therefore, the best possible accuracy of $\mathbf{\hat{\mu}}$ images falls short of generating acceptable $\mathbf{\hat{\varepsilon}}$ images due to the ill conditionality of $\mathbf{M}$. The condition number $\kappa(\mathbf{M})$ correlates to the magnitude of error in the holdup images $\mathbf{\hat{\varepsilon}}$ generated out of it. The use of FBP and EM algorithms to reconstruct ME images in $\mathbf{\hat{\mu}}$ are only expected to exacerbate the low image quality of $\mathbf{\hat{\varepsilon}}$.

![Graph showing condition number vs energy of gamma ray for cases I and II of the phantom.](image)

**Figure 4-7**: Plot of condition number $\kappa(\mathbf{M})$, versus energy of gamma ray for cases I and II of the phantom. This has been generated from the NIST attenuation data for the materials considered (Hubbell and Seltzer 1996). The upper gamma energy has been fixed as that of photon from $^{60}$Co isotope (1332 keV).

The matrix $\mathbf{M}$ is defined by the physical properties of the materials in the phantom and therefore is fixed for the energies of a given set of $\gamma$ photo peaks. The material in the scanned domain can not be changed with the objective of finding a $\mathbf{M}$ with an acceptable condition number as the CT system is supposed to be designed around the flow system. The alternative is for getting accurate $\mathbf{\hat{\varepsilon}}$ images would be to replace the $^{137}$Cs isotope with an isotope that emits $\gamma$ photo peak below 661 keV. Figure 4-7 shows the declining trend in the
\(\kappa(M)\) values for cases I and II of the phantom when the 1332 keV peak (from \(^{60}\text{Co}\)) is used with other peaks of energy lower than it.

Clearly the isotopes that could provide a low \(\kappa(M)\) value would have a \(\gamma\) photo peak with an energy in the range of 150 keV or below, which is far lower than the 661 keV photo peak. However as the profiles in Figure 4-1 indicate, these would have an extremely poor penetration depth (high attenuation) which would lead to a low signal to noise ratio in the counts measured. Therefore, these would not be practical for imaging a domain as large as the phantom in this study or even larger. A similar problem was reported by Bukur et al. (Bukur et al. 1996) in a densitometry based holdup distribution study on a three phase system using the \(^{60}\text{Co}\) and \(^{137}\text{Cs}\) isotopes. Successful studies using the ME approaches where the \(\hat{\mu}\) images have been reconstructed using FBP have been reported (Froystein et al. 2005; Gehrke and Wirth 2005; Hu et al. 2005; Nikitidis et al. 1999; Rapaport et al. 1995; Rebgetz et al. 1991; Rizescu et al. 2001; Yazdi and Esmailnia 2003) for scanners using lower energy (150 keV or below) \(\gamma\) or x photons. The attenuation coefficient at these energies provide lower values of \(\kappa(M)\). Hence the version of (4-5) used for these cases is not too sensitive to the error in the estimates of \(\hat{\mu}\) and thereby they are able to generate \(\hat{\varepsilon}\) images with acceptable error. However, as a trade off they can only be used to image small domains (with an upper bound diameter of about 2 – 3 inches).

Clearly the ME-AM approach even with the use of accurate AM algorithm fails to provide meaningful results for the radio isotopes considered. In this study the simulated transmission data only includes the noise or error associated with the stochastic nature of the \(\gamma\) emission and transmission process. Other sources of noise that could be introduced into the system because of the experimental hard ware and Compton scatter when actual
experiments are conducted have not been included in this study. In spite of this simplification the error in the phase holdup images is extremely high and unacceptable. Hence it is not possible to image phase holdup distribution in three phase systems using the $^{60}$Co and $^{137}$Cs isotopes if the ME-AM approach is followed. If the $^{60}$Co and $^{137}$Cs isotopes are not used than domain larger than 2 inches become impossible to scan when other alternative are used. Clearly a different approach is required to determine the phase holdup distribution.

4.6 Polyenergetic (PE) Approach for Dual $\gamma$ Ray Computed Tomography

The incident photo spectrum of the photons form an x ray source is inherently polyenergetic in nature. Many image reconstruction algorithms related to x ray tomography for medical imaging have been developed that model the polyenergetic nature of the incident x ray photon spectra (Chye Hwang et al. 2000; De Man et al. 2001; Elbakri and Fessler 2002; O'Sullivan and Benac 2007), thereby capturing the physics of the transmission process better. The prime motivation for a model that accounts for the energy of the photons is to reduce beam hardening artifacts that occur in the reconstructed images when a monoenergetic (ME) model is used for spectrum that is polyenergetic (PE) in nature. This artifact appears especially if the domain has high attenuating materials like bone or metallic implants that are typical in medical imaging.

The gamma spectrum consisting of either single or multiple photo peaks (depending on the isotope in question) on the other hand has a discrete spectrum. This allows for the use of energy discrimination electronic hardware associated with the data acquisition system
to acquire photon counts data with a specific energy, even if poor energy resolution
detectors are used. This also helps to avoid counting of the Compton scatter counts
especially if a single discrete gamma photo peak is emitted by the source as in the case of
$^{137}\text{Cs}$. Hence ME models are typically used to model the transmission of $\gamma$ photons
tomography and beam hardening artifacts are rare due to the high energy of the $\gamma$ photons.
In this section the polyenergetic version of the alternating minimization (PE-AM) algorithm
as proposed by O’Sullivan and Benac (O’Sullivan and Benac 2007) is applied to the dual
energy $\gamma$ computed tomography problem

The PE-AM algorithm could be applied such that it accounts for the energy of the $\gamma$
photons and processes the counts data from both the energies of the gamma ray photons.
The notations indicated in Figure 3.1 are used for the discussion in this section. If $\mu(x)$ in
(3-12) represented as

$$q(y : \mu) = I_s(y) \exp \left( - \sum_{x \in X} b(y|x) \mu(x) \right),$$

is replaced with (4-1) and the energy dependant variables are indexed with $E$, the
model for transmission of the $\gamma$ photons can be presented by $q(y, E : \varepsilon)$. Where

$$q(y, E : \varepsilon) = I_s(y, E) \exp \left( - \sum_{x \in X} b(y|x) \sum_{\varepsilon \in \{G, L, S\}} \mu(E) \varepsilon, (x) \right).$$

The I-divergence described n (3-13) can be modified to represent the energy
dependant variable as:

$$I(\|q) = \sum_{E} \sum_{\varepsilon \in Y} \left[ d(y, E) \ln \left( \frac{d(y, E)}{q(y, E : \varepsilon)} \right) - d(y, E) - q(y, E : \varepsilon) \right].$$

The term $q(y, E : \varepsilon)$ is a function of the holdups $\varepsilon_i(x)$, where $i \in \{G, L, S\}$ is the
index for the phases. The over all objective of the algorithm is to minimize (4-11) with
respect to \( \{ \varepsilon_i(x) \} \), the set of holdups of the phases. The overall problem statement is to find the \( \{ \varepsilon_i(x) \} \) that achieve

\[
\min_{\{ \varepsilon_i \}} I(d\|q).
\]

This minimization yields the following iterative update function to estimate the holdup images:

\[
\hat{\varepsilon}_i^{(k+1)}(x) = \hat{\varepsilon}_i^{(k)}(x) - \frac{1}{Z_i(x)} \ln \left( \frac{\tilde{b}_i(x)}{\hat{b}_i^{(k)}(x)} \right),
\]

for \( i = G, L \) and \( S \). Since this study involves an application where data from only two gamma energy peaks are available, (4-13) is used for \( i = L \) and \( S \) alone. For \( i = G \) (4-2) is used in the iterative loop as (4-14):

\[
\hat{\varepsilon}_G^{(k+1)}(x) = 1 - \left( \hat{\varepsilon}_L^{(k+1)}(x) + \hat{\varepsilon}_S^{(k+1)}(x) \right).
\]

As in the monoenergetic case, the terms \( \tilde{b}_i(x) \) and \( \hat{b}_i^{(k)}(x) \) in (4-13) are the back projections of gamma ray counts \( d(y,E) \), and the current estimates of \( \hat{q}^{(k)}(y,E:E) \) over both \( E = 1 \) and \( 2 \) respectively:

\[
\tilde{b}_i(x) = \sum_{y \in Y} \sum_{E} \mu_i(E)b(y|x)d(y,E),
\]

\[
\hat{b}_i^{(k)}(x) = \sum_{y \in Y} \sum_{E} \mu_i(E)b(y|x)\hat{q}^{(k)}(y,E),
\]

where

\[
\hat{q}^{(k)}(y,E:E) = I_s(y,E)\exp\left( -\sum_{x \in X} b(y|x)\sum_{i} \mu_i(E)\hat{\varepsilon}_i^{(k)}(x) \right).
\]
The iterative process was started by choosing a random value for \( \hat{\epsilon}_i^{(k=0)}(x) \) such that \( 0 \leq \hat{\epsilon}_i^{(k=0)}(x) \leq 1 \) for all \( i \) and \( x \) to calculate \( q^{(k)}(y, E : \epsilon) \) in (4-17). Then (4-17) was used to calculate \( \tilde{b}_i(x) \) in (4-16). The back projection \( \tilde{b}_i(x) \) was computed just once using \( d(y, E) \) for \( E = 1 \) and 2. The holdup values \( \{\hat{\epsilon}_i^{(k+1)}(x)\} \) are updated per (4-15) and (4-16), and the process was started again using the updated values of holdup (by setting \( k = k + 1 \)).

A nonnegativity constraint is applied on the values of holdup. Hence \( \hat{\epsilon}_i^{(k+1)}(x) < 0 \) is overwritten as \( \hat{\epsilon}_i^{(k+1)}(x) = 0 \). The scaling factor \( Z_i(x) \) in (4-13) for each \( x \) is chosen such that the following criteria is satisfied (O’Sullivan and Benac 2007):

\[
\sum_i \sum_{x \in X} \delta(y|x) \mu_i(E) \frac{1}{Z_i(x)} \leq 1. \tag{4-18}
\]

In this case, the product of the length of the longest projection though a pixel and the attenuation of the phase at the highest energy of \( \gamma \) was chosen as \( Z_i(x) \) value. Equations (4-13) and (4-14) were used to determine the holdup distribution images of the three phases in the system.

If other radio isotopes are used as the source for \( \gamma \) photons such that it emits more than two energy peaks of \( \gamma \) photons then in theory the PEAM model can be applied to such cases. If transmission data from \( n \) energies of \( \gamma \) photons is available then \( n + 1 \) phase holdups can be calculated using the PEAM model. However, even if transmission data from \( n \) or more peaks are available for a three phase system with gas as one of the phases, it still would be advisable to use (4-14) for \( i = G \). This is because equation (4-13) would blow up as the attenuation values \( \mu_{c_i}(E) \) gasses have at the energies typical of gamma rays are very small (almost equal to zero).
This application of the PEAM algorithm can be viewed as a special case of the double minimization $\min_p \min_q I(p||q)$ (O'Sullivan and Benac 2007). Since the counts data in this study are assumed to be energy discriminated (measured as $d(\gamma, E)$), the algorithm is simpler than the more general alternating minimization algorithm proposed by O'Sullivan and Benac (O'Sullivan and Benac 2007).

The PE-AM algorithm discussed in this section was applied to the transmission data from cases I and II of the phantom in Figure 4-2.

### 4.7 Results of Polyenergetic Approach (PE-AM)

The same simulated transmission processed by the ME-AM approach discussed in section 4.4 was processed using the PE-AM algorithm. The results obtained using (4-13) and (4-14) have been analyzed and discussed in this section. $5 \cdot 10^4$ iterations were carried out to ensure the I-divergence is minimized for this case. For comparison of the results a uniform color scale similar to those associated with images in section 4.5 have been used. Phase holdup images reconstructed with PE-AM are shown in Figure 4-8 for the cases I and II of the phantom. The first row depicts the truth holdup images, the second and third rows show the holdup images for the cases I and II of the phantom respectively. The column A, B, and C represent the gas, liquid and solid phases respectively.

The holdup images obtained using PE-AM are far clearer with fewer artifacts as compared to the ME-AM holdup images (Figure 4-5). The margins defining the shapes of the subdomains C1 through C4 are clearly visible and resemble the truth (Figure 4-2). The section plots comparing the holdup truths with the PE-AM holdup results for the three phases are shown in Figure 4-9. For both cases I and II of the phantom, the PE-AM
reconstructed phase holdup image profile closely follows the truth profiles. The holdup values at no point are greater than 1 or less than 0 as noticed in the ME-AM case (Figure 4-6). These results show that the PE-AM is able to quantify the phase holdup distribution very accurately and provide meaningful results, even for a very ill conditioned case II.

The PE-AM approach incorporates the linear constitutive equation relating the mixed flow’s attenuation coefficient to the phase holdups given by (4-1) directly into the model for transmission of gamma ray photons (4-10). The phase holdup images are then computed based on the iterative expression obtained by minimizing the I-divergence. Since the holdup images are directly iterated, the use of intermediate images and (4-5) which depends on an ill conditioned \( M \) (4-4 or 4-6), is obviated. Also, since data from both the energies are used simultaneously, the data pool available to the PE-AM for iteration is doubled compared to that used for reconstructing the attenuation coefficient images \( \hat{\mu} \) in ME-AM.
Figure 4-8: Holdup image reconstructed using PE-AM approach. Case I phantom with lead glass as the solid phase and Case II of the phantom pyrex glass as solid phase. Columns; A: gas holdup, B: liquid holdup and C: solid holdup.
The fundamental change of directly using the constitutive equation (4-1) in the reconstruction process directly versus using it post processing makes a remarkable difference in the quality of the results. The results are of high qualitative and quantitative accuracy.

4.7.1 Convergence Characteristics of PE-AM

The PE-AM algorithm gives highly accurate results with little sensitivity to the condition number of the \( M \) matrix associated with it. It takes a high number of iterations to minimize the I-divergence. This section analyzes the characteristics of the PE-AM algorithm.
Since the PE-AM algorithm minimizes the I-divergence, the properties of the algorithm are dependent on the characteristics of the I-divergence. To analyze it better, a simple phantom as shown in Figure 4-10 was considered. This phantom has a uniform composition such that the main domain B and the subdomains C1 – C4 have a common composition that is the equal to composition of subdomain C2 ($\varepsilon_c = 0.6, \varepsilon_l = 0.1$ and $\varepsilon_s = 0.3$) from Table 4.1. This phantom is such that the global and the local values of gas, liquid and solid holdups are the same.

![Figure 4-10: Case of the phantom with a uniform mixed composition of the phases in the region B. The composition is the same as the subdomain C2 of the phantom in Figure 4-2.](image)

If the constitutive equation (4-1) representing the attenuation coefficient of the mixed multiphase flow, is parameterized by the parameters $\gamma_L$ and $\gamma_S$ which represent all possible values of holdups $\varepsilon_L(x)$ and $\varepsilon_S(x)$ in the range $0 \leq \varepsilon_i \leq 1$ for $i \in \{L,S\}$, then the attenuation of the phantom could be represented as:

$$\mu(E) = \mu_L(E)\gamma_L + \mu_S(E)\gamma_S.$$  \hspace{1cm} 4-19

The gas phase is not included here as the values of $\mu_g(E)$ for $E = 1,2$ are negligible (Table 4.2). Equation (4-10) could be rewritten as
\[ q(y, E : \gamma) = I_s(y, E) \exp \left( -H(y) \sum_{i \in \{L,S\}} \mu_i(E)\gamma_i \right) \]  

4-20

Since the local holdup values, which are usually indexed by \( x \), are the same as the global values the holdup parameters \( \gamma_L \) and \( \gamma_S \) are spatially invariant and thereby are not indexed by \( x \) in (4-20). \( H(y) \) in (4-20) represents the length of the projection across domain, this is the same as the sum of the segments \( b(y|x) \) of the projection \( y \) in all the pixels along the path of projection \( y \). This can be represented as:

\[ H(y) = \sum_{x \in X} b(y|x). \]  

4-21

Hence when compared to (4-10) there are no \( x \) dependant terms in (4-20). The I-divergence (4-11) for the parametric model \( q(y, E : \gamma) \) can be written as written as:

\[ I(d(y) \| q(y : \gamma)) = \sum_{E} \sum_{y \in Y} \left[ d(y, E) \ln \left( \frac{d(y, E)}{q(y, E : \gamma)} \right) - d(y, E) - q(y, E : \gamma) \right], \]  

4-22

where \( \gamma_i : i \in \{L,S\} \). The simulated detector counts \( d(y, E) \) have been generated based on the truth holdups \((\epsilon_L = 0.6, \epsilon_L = 0.1 \) and \( \epsilon_S = 0.3 \)) based on the procedure described in section 3.1.2. The minimum of (4-22), \( \min I(d(y) \| q(y : \gamma)) \), exists when \( \gamma_L = \epsilon_L \) and \( \gamma_S = \epsilon_S \).

Figure 4-11 shows a contour of the values of \( I(d(y) \| q(y : \gamma)) \) calculated form (4-22), for the range \( 0 \leq \gamma_i \leq 1 \) of the parameters \( \gamma_i : i \in \{L,S\} \). This contour of \( I(d(y) \| q(y : \gamma)) \) has a minimum at \( \gamma_L = 0.1 \) and \( \gamma_S = 0.3 \) which are the truth values of \( \epsilon_L \) and \( \epsilon_S \) used. Since the composition of each pixel in the domain is equal, if the \( I(d(y) \| q(y : \gamma)) \) contour for every pixel were to be written it would be the same. Hence the
local and the global contour is the same as the local and global holdups are the equal. I-divergence represents a quantity that is summed over all the projections \( y \) that pass through all the pixels \( x \). If the each pixel has different phase holdups then it would be hard to parameterized all of them in terms of \( \gamma_L \) and \( \gamma_S \). An I-divergence contour for a more complex phantom like the one in Figure 4-2 would be difficult to plot with a \( \gamma_L \) and \( \gamma_S \) as axis. Hence the contour for a simplified case of the phantom shown in Figure 4-10 has been plotted for the purpose of analysis.

![Contour of I-divergence](image)

**Figure 4-11:** Contour of I-divergence -- the cost function used to determine the holdup values. This contour has been plotted for case I of the phantom if all the domains (B and C1 – C4) were of the same composition as subdomain C2.

The I-divergence contour, as Figure 4-11 indicates, has a steep descent for \( I(d\|\theta) \) along the \( \gamma_s \) axis that converges to the point on the \( \gamma_s \) coordinate where \( \gamma_s = \varepsilon_s \). The gradient along the \( \gamma_L \) axis is not as sharp as the gradient along the \( \gamma_S \) axis. The contour could be compared to two ‘mountain sides’ represented by the gradient along the \( \gamma_S \) axis, enclosing
a ‘valley’ along the $\gamma_L$ axis. This asymmetry in the gradient suggests that for any initial
guess $\hat{e}_i(x)$ for $i \in \{L,S\}$, the $\hat{e}_L(x)$ image reaches closer to the truth faster than
the $\hat{e}_L(x)$ image. If an iteration is started with an initial guess $\hat{e}_L(x) = 1$ and $\hat{e}_S(x) = 1$,
the algorithm would move down the ‘mountain’ along the $\gamma_L$ axis with high rate of change in
the value of the $I(\|D\|)$ per iteration. Once the algorithm is in the ‘valley’, the bias in the
$\hat{e}_L(x)$ image would be far less than the bias in the $\hat{e}_L(x)$ image. The algorithm would then
show a lower rate of change in the values of $I(\|D\|)$ thereby requiring many more iterations
to move along the $\gamma_L$ axis to reach $\min I(\|D\|)$ thereby reducing the bias in $\hat{e}_L(x)$. In other
words, an initial guess $\hat{e}_L(x) = 0.12$ closer to the truth, and $\hat{e}_S(x) = 1$ furthest from the
truth, would rapidly converge compared to the initial guesses the other way around
with $\hat{e}_L(x) = 1$ and $\hat{e}_S(x) = 0.32$. Hence the phase with a lower value of the
attenuation constant controls the dynamics of the iterative process of the PE-AM.

Figure 4-12: Plot of I-divergence versus iterations for different set of initial guesses values
of the hold ups for cases I and II of the phantom
The PE-AM iterative dynamics were similar when the images in Figure 4-8 were reconstructed based on the sinograms shown in Figure 4-3. Figure 4-12 shows the plot of current values of I-divergence versus the iterations for the cases I and II of the phantom calculated using (4-11). When the initial guess condition A – with \( \hat{\varepsilon}_L^{(k=0)}(x) = 0.57 \) and \( \hat{\varepsilon}_S^{(k=0)}(x) = 0.08 \), were found to be favorable (Figure 4-12). This is attributed to the value of \( \hat{\varepsilon}_L^{(k=0)}(x) = 0.57 \) being closer to the truth value \( \varepsilon_L(x) = 0.6 \) in the domain B (Figure 4-2), which constitutes the bulk of the area in the phantom. The I-divergence descends rapidly and monotonically until \( k = 10^2 \). The rate of descent then declines as initial guess values are unfavorable for the subdomains C1 – C4. The \( \hat{\varepsilon}_L^{(k=100)}(x) \) images for cases I and II (not shown here) show the region B with a much lower bias than subdomains C1 – C4.

Eventually all the subdomains converge in the \( k = 3 \times 10^4 – 5 \times 10^4 \) range. If the subdomains C1 – C4 were absent the I-divergence would continue to drop monotonically and the images would converge in the \( k = 10^3 \) range. The initial guess condition – B with \( \hat{\varepsilon}_L^{(k=0)}(x) = 0.26 \) and \( \hat{\varepsilon}_S^{(k=0)}(x) = 0.80 \), was found to be unfavorable. The rate of descent in I-divergence values is far less than condition A. The curves reach a plateau in the \( k = 10^4 – 5 \times 10^2 \) range almost suggesting convergence. Since the initial guesses are unfavorable for subdomains C1 – C4, the images eventually converge in the \( k = 3 \times 10^4 – 5 \times 10^4 \) range. The absence of the subdomains would not hasten the convergence. Hence, for a large number of iterations the PE-AM ultimately converges and does not have any initial guess dependency.
4.8 Conclusions

The results show that the ME approach (typical of what is followed in literature), to the dual energy gamma ray tomography problem, does not perform well for the choice of the energy of the $\gamma$ photons considered. The ME-AM algorithm is known to provide accurate $\hat{\mu}$ images (Varma et al. 2008). This accuracy is not enough to determine the holdup images using post processing due to the high condition number of the matrix that relates $\hat{\mu}$ to $\hat{\varepsilon}$. It is this limitation that has restricted its use to small flow domains with lower energy gamma photons and has hindered the development of high energy gamma ray scanners for large scale systems as it is impossible to use high energy gamma ray photons with ME algorithms.

The PE-AM algorithm when applied to the dual energy $\gamma$ ray tomography problem provides high quality images that accurately quantify phase holdup distribution without any external constraints or the application of priors or regularization methods. The results are not limited by the attenuation characteristics of the materials that constitute the phases. The high computation cost of the algorithm is not an issue when it is used for processing data offline. There is scope to exploit the properties of the I-divergence contour to evolve strategies for faster convergence. Since the initial guess for phase with the lower attenuation (which is liquid in this case) determines the iteration dynamics, smart choices of initial guess values lead to faster convergence. If in a multiphase system the liquid phase is continuous, the global liquid holdup could be used as the initial guess versus a random number for the iterative process.

The application of PE-AM shown in this chapter provides bases for processing dual energy gamma ray transmission data for a wide range of energies to determine phase holdup
distribution in multiphase flow systems. Scanners using high energy gamma ray sources could be developed as a research or diagnostic tools for imaging phase holdup distribution in large scale three phase systems on the basis of this algorithm. The PE-AM algorithm opens doors for experimental research in the area of three phase flow that would provide better understanding of phase interaction in a wide range of multiphase systems.
Chapter 5

Development of Dual Energy – Dual Source Computed Tomography system

5.0 Introduction

Single source γ ray computed tomography (SSCT) systems that used a single γ energy photon of 661 keV from $^{137}$Cs isotope have been developed at the Chemical Reaction Engineering Laboratory (CREL). The first system in CREL was developed by Kumar (Kumar 1994). This system has a fan beam arrangement of source and detectors with a fan angle of 40°. The system collects 77 projections in one source position or view using 11 detectors. The system collects projection data for a total of 99 views. The expectation maximization (EM) discussed in Chapter 3 was implemented for image reconstruction and attenuation images were used to determine the phase holdup distribution. The Kumar (Kumar 1994) SSCT system is capable of imaging column with a maximum diameter of 18 in. The second SSCT in CREL was developed by Kemoun that operated on similar principles. It was designed to collect 225 projections using 9 detectors per source position. This system was designed to be lighter and portable such that its frame and the data acquisition electronics could be moved to the column or multiphase system to be studied. The system can scan column with a maximum diameter of 12 in and height of 9 ft. The details of design and operational improvements made over Kumar’s SSCT system, and the data processing methodology has been described at length by Roy (Roy 2006a).

The CREL SSCT systems have been validated and used exhaustively to study phase holdup distribution in various two phase system. They have also been applied to study three
phase gas – liquid – solid (GLS) system where the solid phase is stationary (packed beds and structures packing). The SSCT systems cannot be used to study three phase flow systems, a dual energy computed tomography system was developed with intention for use with three phase systems. This dual energy computed tomography system uses the $\gamma$ photons from the $^{137}\text{Cs}$ and $^{60}\text{Co}$ isotopes. Since it has two sources with separate $\gamma$ energies, it has been called the dual energy – dual source computed tomography (DE-DSCT) in this discussion to distinguish it from dual source tomography system DSCT that could use two sources emitting the same $\gamma$ energy.

This chapter discusses the design and development of the DE-DSCT system and its validation with a three phase phantom. It also validates the algorithms discussed in Chapter 4 using the experimental data from the phantoms. Various aspects of the DE-DSCT system including design considerations for safety and operational protocols developed have been included in the discussion.

### 5.1 $\gamma$ ray source

#### 5.1.1 Source selection

It is important to select an appropriate $\gamma$ photon source for the CT system. Along with the attenuation and decay physics and technical considerations a combination of pragmatic considerations such as cost, availability, licensing from regulatory authorities and physical construction influence the process of source selection. The issues related to the source selection discussed in Chapter 4 (section 4.2) are equally valid here. The trends of energy versus attenuation profile show in Figure 4-1 indicate that as the energy of the photon increases the attenuation coefficient of the material reduces.
Froystein (Froystein et al. 2005) and (Van Santen et al. 1995) have discussed criteria for energy selection based on the attenuation characteristics of the material involved and uncertainties in the signal. This would help for the case of x ray tomography as it is possible to control the energy of the x photons. However this is not possible in the case of γ photons as the energy is restricted what the radio isotopes of various elements available in nature (or manmade) emit. Therefore, γ photons for all the energies in the figure may not exists. If a radio isotope does exist that emits γ photons of the desired energy, ready availability onsite for experiments, costs and half life may be a major issue. CREL doesn’t have local access to nuclear reactors that could be used to provide desired radio isotopes as and when desired. Hence the choices are severely restricted. Some of important criteria are as follow:

**Half life and emission factor:** Radio isotopes continuously decay and have a fixed life as the strength or concentration of the isotope exponentially reduces over time. The half life is defined as the time take for its intensity to reduce half of the original. The desired isotope should have a reasonable half life to account for the time required for onsite installation and completion of experiments. Not all decays lead to the desired γ photon. The γ photon of a particular is usually a percentage of the over all decay. This emission factor should be as close as possible to unity.

**Cost and commercial availability:** For obvious reasons the cost of the isotope should be low, including disposal costs when it has decayed completely. The cost is to be weight against the half life of the source. It should be available commercially in a sealed double layer steel encapsulation possibly from multiple suppliers/vendors.

**Physical shape:** The fan beam configuration of CT requires the source to be in a point form to get images that are not blurred. In practice it is bound to have a finite
dimension, hence the source of the steel capsule it is enclosed in, should be as small as possible and as close to a sphere in a physical form as possible.

Table 5-1: List of candidate radio isotopes (Ledere et al. 1966)

<table>
<thead>
<tr>
<th>Radio Isotope</th>
<th>Half life</th>
<th>Energy of $\gamma$ photon (keV)</th>
<th>$\gamma$ emission factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{241}$Am</td>
<td>421 years</td>
<td>66</td>
<td>0.38</td>
</tr>
<tr>
<td>$^{169}$Yb</td>
<td>32 days</td>
<td>63, 109, 130</td>
<td>-</td>
</tr>
<tr>
<td>$^{109}$Cd</td>
<td>1.27 years</td>
<td>88</td>
<td>0.04</td>
</tr>
<tr>
<td>$^{153}$Gd</td>
<td>241 days</td>
<td>98</td>
<td>0.55</td>
</tr>
<tr>
<td>$^{57}$Co</td>
<td>271 days</td>
<td>122</td>
<td>0.86</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>30 years</td>
<td>661</td>
<td>0.85</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>5.24 years</td>
<td>1173, 1332</td>
<td>1.0</td>
</tr>
<tr>
<td>$^{75}$Se</td>
<td>120 days</td>
<td>121, 136, 264, 279, 401</td>
<td>0.17, 0.58, 0.57, 0.25, 0.11</td>
</tr>
</tbody>
</table>

Based on these criteria the sources listed in Table 5-1 were indentified. The $^{169}$Yb has a half life that is impractical. Although the half life of $^{241}$Am is not an issue, the energy of the $\gamma$ photons is very low. Due to nonlinearity of attenuation with energy for a fixed domain size of say water, the signal obtained from 0.1 Ci $^{137}$Cs would require a 7-8 Ci $^{241}$Am source. These created a lot of health hazards if one is accidently exposed the source at a close range. Also, the $^{241}$Am has a self absorbing or attenuating property attributed to its high atomic number, due to which its not possible to make point sources beyond 200 mCi strength. Considering half life, energy such that the $\gamma$ photon would not be totally attenuation, and cost, $^{137}$Cs and $^{60}$Co are the appropriate choices. The $^{137}$Cs is the source of choice for nuclear gauging applications in the industry as it has a long half life and emits single clean peak and
don’t have scatter interference of photons of other energies. $^{60}$Co is the next best as the signal from the highest peak would not have any scatter interference. Since the second peak is close to the first, the scatter interference in signal is minimal. These two sources are very widely used in a lot of industrial related nuclear applications and can easily be sourced from vendors.

A second choice could be $^{75}$Se as it has the advantage of using four peaks (121 though 279 keV) from a single source which would provide redundancy in the data collected. The overall cost would be reduced as two sources are not required and a single fan beam arrangement of source and detectors could be used. However the $^{75}$Se require much higher strength than $^{137}$Cs and $^{60}$Co sources. Therefore the operation protocol would need to be more stringent as certain areas of the DE-DSCT setup would have a very high radiation exposure.

### 5.1.2 Source collimator device

The point source is housed in a shielded container, a part of which is opened when the gamma beam is required for the CT experiments. This device selectively collimates the beam to give it a fan shape. Hence it’s called the source collimator device (SCD) in this discussion. For the DE-DSCT setup, two such devices are required as two sources are used. Figure 5-1 shows the photograph of the SCD used. The SCDs were designed and fabricated by Dr. Charles Alexander at the Oakridge national laboratory (ORNL). Details of the source location in the SCD, the operation protocol, the machines drawings giving details of the dimensions and some additional photographs related to the SCDs are available in Appendix-I. The point source is located in the center of the SCD. When a γ beam is required the window wedge is removed, the top plate is opened and the source is remotely lowered such
that t is aligned with the wedge. A radiation from the source get collimated to fan beam with a 40° angle and 5 mm height as it emerges from the window in the SCD.

Figure 5-1: Photograph of the SCD made of tungsten used for $^{60}$Co. The $^{137}$Cs SCD is made of lead and is similar to the $^{60}$Co SCD.

Since the $^{60}$Co source emits very high energy $\gamma$ photons (see Table 5-1), the SCD used for it is made of tungsten. Tungsten has a high density (~18 gm/cc) hence it is able to shield the $^{60}$Co better. The SCD for the $^{137}$Cs source is made of lead, as the lead is sufficient for shielding the $\gamma$ $^{137}$Cs source. When the SCD are placed on the DE-DSCT setup, some additional paraphernalia are included. These include an external beam collimator to further collimate the height of the beam to keep it narrow, and a collimator strap secured that secures the SCD on to the DE-DSCT (details discussed in Appendix-I).
5.2 The Dual Energy-Dual Source Computed Tomography Setup

This section of the chapter discusses the details of the dual energy-dual source computed tomography setup (DE-DSCT) setup. The details of the hardware, the electronics data acquisition electronics setup and operation are presented.

5.2.1 Description of the DE – DSCT setup

Figure 5-2: Schematic of the DE-DSCT showing the top view. The red region in the center between the SCD and the detector array indicates the γ ray beam (physically not visible)

Since two separate sources are used, the DE – DSCT setup consists of two separate fan beam arrangement of source and detectors each dedicated to one source. Figure 5-2
shows a schematic of the DE-DSCT setup showing the top view. The black circular region in the middle with a diameter of 30 inches is and open space where the multiphase systems to be scanned is placed such that it is simultaneously exposed to both the \( \gamma \) beams. The shaded conical sections over this (shown in red) indicate the \( \gamma \) beam emerging from the SCDs. A detector array is located at the side opposite to each SCD in the respective fan beams. These arrays count the unattenuated photons (or even the scattered photons) of the \( \gamma \) ray that pass through the multiphase experimental setup. Figure 5-3 shows a photograph of the top view of the actual DE-DSCT setup represented in the schematic in Figure 5-2. The detector array pates and the SCDs are placed on the circular source plate. This circular source plate is connected to the base plate through a circular rack and pinion arrangement (not shown). The rack is mounted on a series of bearing along the edge. A stepper motor is connected to the pinion (not shown) that moves the rack which in turn angularly moves the circular plate. The base plate remained fixed.

Figure 5-3: Photograph showing the top view of the DE-DSCT setup.

The base plate is connected to four vertical ball screws that are connected at the upper and lower ends the aluminum frame (shown in Figure 5-4 and not shown in Figure...
This ball screws have pinion in the bottom that is connected to a common chain. This chain in again connected to a motor with a pinion, the motor moves the chain which in turn rotates all the ball screws in a synchronous manner. This way the base plate connected to the ball screws can be moved to different elevation along the vertical axis of the setup. The motor is run clock wise or counter clock wise to move the base plate up or down, to adjust the height of the base plate to the desired level. The total clearance of the ball screw is about 10 ft. Hence the DE-DSCT can scan columns a little under 9 ft in height (leaving some space to accommodate the SCDs) and 30 inches in diameter.

Figure 5-4: Photograph of the DE-DSCT setup showing the front view of the system
Figure 5-5: Schematic of the fan beam arrangement of the detector collimators with the Detector array shield. A view of section A-A’ is shown in Figure 5-7.

Figure 5-5 shows the schematic of the fan beam arrangement of source and detectors used in the DE-DSCT setup. A single fan beam has been shown in the figure. The detectors are placed behind an arch shaped monolithic detector array shield made of lead which has a 2.5 inches thickness, 3 inches height, and a 45 inch radius with the location of the source in the SCD as a reference. The detector shield has 15 wholes of 1 inch diameter bored in to it such that it is able to accommodate lead collimators with a fine aperture as shown in Figure 5-6. The angle between the central axis of successive detectors within the fan is 2.743° using the location of the source as the reference. The total angle between the central axis of the 1st and the 15th detectors in the fan beam is 38.402°. The fan beam can
encompass a column with an 18 inch diameter when placed in the center of the DE-DSCT setup (Figure 5-2). The $\gamma$ fan beam has an angle of 42°, hence it covers all the detectors with in the fan. The dotted lines passing through the detectors collimator apertures that joins the source and the detectors, represents the path of the projection or the line along with the transmission measurement of the $\gamma$ photons is made across the domain. This is also the line of sight between the detector and the point source that passes through the detector collimator.

Figure 5-6: Schematic of the detector collimators that are inserted into the detector array lead shield.

Figure 5-7 shows the vertical elevation of section A-A’ as indicated in Figure 5-5. The outline of the SCD (Figure 5-1) is indicated in Figure 5-5. The point source is within the SCD as shown. The collimation by the SCD and the additional source beam collimator (visible in Figure 5-3), helps control the height of the emerging $\gamma$ ray beam such that is less than the height of the detector array shield. Only the portion that passes through the detector collimator’s aperture is what is recorded by the detector. The detector assembly has
a cylindrical NaI(Tl) crystal with a 2 inch length and a 2 in diameter which detects the photons. Only the crystal segment I shown in Figure 5-7. To block the unattenuated or unabsorbed photons that pass through the detector system small pieces of lead that 1 inch thick are used as beam stops. These beam stops are propped by a mechanism (not shown in Figure 5-7; called out in Figure 5-8) such that they are axially aligned to the aperture of the detector collimator. The beam stops are an additional safety component.

Figure 5-7: Section A-A’ (Figure 5-5) view of the fan beam showing the SCD and detector arrangement. The shaded triangular region in the middle (indicated in red) shows a vertical section of the gamma ray fan beam.

The entire assembly of the detector array, which includes the detector array shield with the collimators inserted in them, the detector assembly and the beam stops with the support prop, are mounted in top of a plate (called out in Figure 5-3). This plate also has a curvature like the detector array shield and is mounted with rollers on top of the circular source plate (indicated in Figure 5-2 and called out in Figure 5-3). Figure 5-8 shows the detector array plate with stepper motors anchored on top with the aid of support mechanism
that helps move the detector assembly. The motor shaft is attached using a coupling to that shaft of a ball screw with a fine pitch such that one end of the ball screw shaft is free (see Figure 5-8). The nut associated with the ball screw assembly, that encompasses the shaft, is fixed onto a mechanism that in anchored on to the circular source plate. Hence when the motor moves, the detector array plate also moves sideways along with it in the direction of the arc of the detector fan.

![Figure 5-8: A photograph of the stepper motor and mechanism associated with the detector array motion for both the fan beams](image)

### 5.2.2 Description of data acquisition system

The data acquisition system consists of the electronic hardware that links the detection of the \( \gamma \) photons to ultimately the photons counts registers as numbers with in the computer. This consists of the detectors, preamplifier, pulse processors and stepper motors that automate the motions involved in the DE-DSCT system. This system is similar to the
one used in MP – CARPT system developed at CREL. Since it has been described at length by Vesvikar (Vesvikar 2006), only key elements have been revisited here for the sake of continuity and the additional components included for automation of the DE – DSCT setup have been discussed.

The choice for the detector was based on a trade of between the cost and the absorption efficiency. A detector assembly consisting of a 2 in × 2in NaI crystal connected to a photomultiplier tube followed by preamplifier was used. The detectors made by Saint-Gobain Crystals sold with brand name Bicron were used. Although other detectors such as the CdWO₄, BGO etc have a better performance was compared to NaI, it was chosen as it is far cheaper than other options (Saint-Gobain 2004). A large size crystal was used as smaller sizes of the crystal have poorer absorption efficiency for higher energy of the photons. A 2 in × 2in crystal has about 70% absorption efficiency for ¹³⁷Cs and about 50% for ⁶⁰Co sources (Saint-Gobain 2004). The Canberra preamplifier was used as the bas for the detector.

Figure 5-9 shows the schematic of the electronics system. The detectors are powered by a Canberra power supply. The signal from the preamplifier, attached to the detector, is fed to the timing amplifier. This is in turn passed on the multilevel discriminator and a scaler. The final data is stored in the computer in form of a data file. The electronic modules associated with the NIM bin crate and the PCI crate was developed by Dr. Alan Winternberg at the oakridge national laboratory (ORNL). A C++ based program called Biomass.c developed by Lloyd Clonts under the supervision of Dr. Alan Winternberg was passed on to CREL. The working details of the data acquisition modules developed at ORNL, cost advantage over the first generation CREL system, and the commands and
operation of *Biomass.c* have been discussed at length by Vesvikar (Vesvikar 2006) (see Appendix B of Vesvikar, 2006) and have not been repeated here.

Figure 5-9: Schematic of the data acquisition system used in the DE-DSCT

A very useful feature of this system is that parameters such the sample time, sample frequency etc can be specified in an input text file. *Biomass.c* uses those values to run the system and the counts data recorded is outputted into a user specified data file. A subroutine
called *Commrun.c* that passes on relevant commands for the motion of the stepper motors to the motor controller through the com port was developed. *Commrun.c* is called from the DE-DSCT version of the *Biomass.c* to move the motors associated with the DE-DSCT. The input parameter for the motion of the motors is stored in a text file such that *Commrun.c* reads the values and executes the motor motion. The computer is connected to a set of motor controllers (one for each stepper motor) that are wired in a daisy chain configuration. The commands from the computer move the detector plates and the source plate to a precise distance. The system was programmed to wait for a feed back from the motor controller when *Commrun.c* was called in *Biomass.c* confirming the completion of motion before the counts data from the detector is recorded.

A key feature of this system is that it can be operated in two modes: the fine and the coarse. In the fine mode the system collects spectral data that is discriminated into 1024 bins such that each bin represents any where between 0.5 – 2 keV depending on the gain of the and the voltage setting of the detector. In the coarse mode the user can specify eight windows based on the fine mode as the reference with a lower and an upper limit. The area under the spectral curve (obtained from fine mode), or the integral, between the lower and upper limits of energy (specified in terms of bin numbers) is provided in the readout. Based on the ORNL hardware modules, *Biomass.c* is programmed to records the counts of photons up to eight different energies or peaks and provide separate readouts. The spectra of $^{60}$Co and $^{137}$Cs sources obtained from a singe detector in each of the fan beams by running *Biomass.c* in the fine mode is shown in Figure 5-10. Since the fan beams are well collimated and two separate sources are used with independent set of detectors, the scatter from the higher energy photons of $^{60}$Co doesn’t appear in region beyond the peak in the $^{137}$Cs spectrum. If the energy windows is set by specifying the low and high bin numbers before
and after the peak respectively, as indicated in Figure 5-10, the counts of the unattenuated photons, which the area under the segment of the curve between the high and low bin can be recorded. This high and low bin specification also ensures that the photon counts due to Compton scatter are not included.

![Figure 5-10: Incident photo spectra of the counts from the $^{60}$Co and $^{137}$Cs sources as seen by the Bicron NaI detector. The data was collected for 2 s.](image)

### 5.2.3 Working of DE-DSCT setup

The multiphase system to be studies in placed in the center of the DE-DSCT setup. This region is called out in Figure 5-3 and is also the black region Figure 5-2. The base plate of the DE-DSCT is set at the elevation where the multiphase system is to be imaged. The data acquisition program is run in the fine mode at a user specified sample time. The data from the output file is plotted to determine the location of the energy peaks for each detector. The bin location of the peak could vary from detector to detector depending on its individual gain or tuning. The low and high bin numbers such that peak of interest is included, are noted and fed as input to DE-DSCT version of Biomass.e when it’s run in the coarse mode. The output data file contains the counts data of the unattenuated photos with
unpolluted by scatter counts. The program can accept up to 8 such ranges (low and high) of bin numbers for a given channel (or detector). The user also needs to specify the number of data samples to be collected for each projection, and the frequency at which the data samples are collected for all the detectors.

The program then moves the stepper motors attached with the detector array plates associated with the two fan beams Figure 5-8. The detector array is moved clockwise to the starting position by an angle of 1.3° by the stepper motor connected to it, as indicated in the Figure 5-11A. After this the detector array is moved counter clockwise in 20 times in small 0.13° angular steps. After the motor motion for each step is completed for each step of both the arrays, the photons counts data is recorded as per the input parameters specified to Biomass.c. This was data is collected for 21 projections (including the starting and the end position) by each detector. A total angular sweep 2.6° is covered by for each detector, taking the total angular sweep of the complete fan beam to 41.002°. The final position of the detector array is in the extreme right, which is 1.3° counter clockwise from the central position as shown in Figure 5-11B. This movement help a fan beam with 15 collimated detectors get 315 (15 × 21) spatially independent equiangular (with respect to the location of the source) projection measurements. Hence, the motion of the detector converts a fan beam of 15 detectors crystals of 2 inch × 2 inch cylindrical dimension into a fan beam with 315 detectors with effective crystals cross section of 1/16 inch × 3/16 inch and length 2 inch. The reduction in the effective open area of the detectors is of course due to aperture of the lead collimators (Figure 5-6) placed in front of the detectors with in the detector array.
Once detector array motion is completed, the detector arrays move back to the starting position again. The circular source plate rotates a small angle with respect to its center such that SCDs that house the sources, are oriented to a new position along the circular locus called out in the DE-DSCT schematic in Figure 5-2. This motion is accomplished by the stepper motor connected through a pinion to the circular rake attached to the circular source plate. Usually this movement is about 1.84° with respect to the central axis of the DE-DSCT setup. The detector fan motion is repeated again to effectively obtain 315 projection measurements for the new position. The source plate is moved 196 times to get projection measurements for 197 source positions. Each source position for which projection measurements are made is called a view. Hence during one scan a complete 360°
rotation of the source is done around the multiphase system in the center of the DE-DSCT setup. Hence with 197 views and 315 projections per view, a total of 62,055 \((197 \times 315)\) spatially independent projections measurements are made across the domain during one experiment. This process usually takes about 6-7 hrs. It could be increased a little by increasing the speed settings of the motor. Hence the image generated based on the projections data collected over a long period of time provides is a time averaged phase holdup distribution in the domain. The Biomass.C can be programmed to change the number of views as per the requirement of the experiment.

### 5.2.4 Radiation safety considerations

The DE-DSCT system uses a 250 mCi \(^{137}\text{Cs}\) isotope and a 50 mCi \(^{60}\text{Co}\) sources in a sealed form. The SCDs that house these source are very well designed such that source is locked into it at all times and cannot be inadvertently separated from the SCD. The contact dose rates are 3 mr/hr and 5 mr/hr for the \(^{137}\text{Cs}\) and \(^{60}\text{Co}\) SCDs respectively. Once the wedge is removed to allow the \(\gamma\) beams to emerge, the high radiation area is confined to the center of the DE-DSCT setup. The detector array lead shield and beams stops minimize the occupancy dose around the DE-DSCT setup. The dose rate at a perimeter of 2 ft from the DE-DSCT setup is less than 1 mr/hr when measure directly inline with the fan beam. The operational protocols of the DE-DSCT have been thoroughly vetted by the Radiation Safety Officer (RSO) at Washington University. The dose rates around the setup due to the operations are monitored constantly by the RSO. Also the DE-DSCT lab is regularly inspected by the RSO to monitor compliance with protocols. The details of the dose rates at different location in and around the setup DE_DSCT setup, the operational protocols and procedures developed for this system have been discussed in Appendix – I. This document (Appendix-I) has been vetted and approved by the by the RSO and is the basis for all DE-
DSCT operations. The setup is safe to use if all the protocols are followed in letter and
sprint. The source are sealed and are housed in the SCDs, hence leaks are not possible.

5.3 Validation of DE – DSCT setup

This discussion in the previous sections has focused in the working of the DE-
DSCT setup and details of the paraphernalia associated with it. In this section the validation
of the DE-DSCT setup and the PE-AM algorithm discussed in Chapter 4, has been carried
out. Experimental projections data of a three phase phantom from a scan using the DE-
DSCT setup has been used for the validation.

5.3.1 Details of the three phase phantom

The DE-DSCT setup is intended to image phase holdup distribution in three phase
systems. Therefore to validate it, a phantom with three phases: gas – liquid – solid was built.
The schematic top view is shown in Figure 5-12A. The outer rim of the phantom was built using a 10 inch diameter Perspex tube with a 2 inch height by mounting (with glue) on a flat Perspex plate. Two subdomains C1 and C2 are carved out using Perspex tubes 2 inch diameter and 2 inch height. Three circular pieces of solid glass 2 inch thick cut into circles was used as the solids subdomain S1 – S3. Lead glass supplied by Technical Glass Products which has a density of density of 4.5 gm/cc was chosen as the solid phase. The dimensions details of the different components of the phantom are shown in Figure 5-12B. Two configuration of the three phase phantom were considered, Case I and Case II. The details are available in Table 5-2. The idea is to interchange the liquid and the gas phases while keeping the solid phase in the same location.

<table>
<thead>
<tr>
<th>Phantom</th>
<th>Substance in region B</th>
<th>Substance in subdomains C1 and C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Case II</td>
<td>Air</td>
<td>Water</td>
</tr>
</tbody>
</table>

### 5.3.2 DE-DSCT scan of the phantom

The base plate of the DE-DSCT system was aligned such that it was 1 inch above the base of the phantom. Since the diameter of the phantom is 10 inches, only 9 detectors in the middle of each of the detector array (omitting three on the left and right) are sufficient such that portion of the fan beam covered by them encompasses the phantom. Hence, for all scans described in this section 197 views (source positions) and 189 \((21 \times 9)\) projections
per view are considered. Thus a total of 37233 (197 views × 189 projections) projection measurements were made across the 10 in domain of the phantom. The counts for each projection were collected at 10 hz. To improve the accuracy of the data, for each projection measurement 80 data samples were collected at 10 hz. Then (3-22) was used with $n = 80$ to determine the mean of the projection measurement. This mean value of the data is used to construct the sinograms for cases I and II as shown in Figure 5-13.

![Figure 5-13](image)

**Figure 5-13:** Sinograms from the scan of the three phase phantom (Figure 5-12). A and C; show the sonograms of the 1332 keV $\gamma$ photo peak from $^{60}$Co source for cases I and II (Table 5-2) of the phantom. B and D; show the sonograms of the 661 keV photo peak from $^{137}$Cs source, for cases I and II of the phantom.

The incident photons counts $I_y(E)$ were estimated with the empty phantom (only the Perspex shell) in position. Hence, the measured projections data $I_y(E)$ represents the
attenuation due to the material in the phantom and omits the attenuation due to the walls. The transmission ratio

\[ T(y, E) = \frac{I(y, E)}{I_s(y, E)}, \tag{5-1} \]

was calculated using the measure incident counts the projection data from the phantom. The sinograms shown in Figure 5-13 for both the cases of the phantom was reconstructed using the transmission calculated from the measured counts. Each pixel in the sonogram represents the transmission ratio for the corresponding projection number given by the y – axis and the view given by the x – axis. The sinograms capture the three circular domains S1 – S3 as the maximum attenuation is by these regions. The region C1 and C2 cannot be seen prominently for case I as water is present in region B. It can however be seen more prominently seen in case II as region B is empty. The sonogram appears to be of good quality as detector artifacts are absent. Artifacts in the form of bands etc point towards a detector anomaly. The sinogram of the 1332 keV peak from the $^{60}$Co (Figure 5-13 A and C) appears to be a little blurred as compared to the sinograms from the 661 keV peak of the $^{137}$Cs source.

5.3.3 Correction of the view number for the $^{60}$Co fan beam

Since the γ energies selected for the DE-DSCT are such that they only available from two separate radioisotopes, the use of two separate detector arrays each dedicated a single isotope become inevitable. These increases the cost of the system as two separate fan beams need to constructed each with it own SCD and detector. However, the increase in cost is offset by advantage of having minimal scatter interference in measurements from the higher energy γ. It is not possible to have both the SCDs at the same position and have the fan beam pass though the scanning domain same level or section. In order to have both the γ
beams simultaneously pass the object at the same level, the SCDs have to place at an angular offset with respect to the center of the DE-DSCT system as indicated in the schematic in Figure 5-2. The angle between the SCDs and by extension between the two fan beams is design to be 115.7° with respect to the center of the column. It was not chosen to be 90° as the dimension of the base plate required to accommodate the two fan would become unmanageable.

If the projections data from the case I phantom is processed to reconstruct separate monoenergetic images, they appear to be spatially dissimilar as shown in Figure 5-14. The monoenergetic images are generated following the procedure described in section 3.1.2 using (3-15) for the data from the two fan beams. The image from the the $^{137}$Cs source (Figure 5-14B) matches the case I phantom image in Figure 5-12, however the image from the $^{60}$Co (Figure 5-14A) doesn’t. This is because the fist view of the $^{60}$Co fan beam sees the phantom from a different spatial orientation than the $^{137}$Cs source. Hence a view correction is required so that the images from the $^{137}$Cs and $^{60}$Co sources match.

Figure 5-14: Monoenergetic imagesof the case I phantom (from Figure 5-12) generated without view correction. A: The monoenergetic image using the $^{60}$Co source. B: The monoenergetic image using the $^{137}$Cs source.
The angular difference of 115.7° between the sources on the circular source plate translates to a gap of 64 views when 197 views are used in total. This view difference between the sources is used to rearrange the projections data array. If the source plate is rotating clockwise with the $^{137}\text{Cs}$ source in the lead followed by the $^{60}\text{Co}$ source, then the physical location of view #1 of the $^{137}\text{Cs}$ source is the same as the physical location of view #64 of the $^{60}\text{Co}$ source. This is because the sources have follow the same circular locus (called out in Figure 5-2) when the source plate is rotated in the DE-DSCT setup. The view number correction can be achieved by storing measured data from the actual view #1 of the $^{60}\text{Co}$ source as view #133 (197 views – 64 views) with reference to the $^{137}\text{Cs}$ source. Hence, when $^{60}\text{Co}$ source is measuring data the at location view #1 of the $^{137}\text{Cs}$ source, the $^{137}\text{Cs}$ source is actually measuring view #64. The view number correction is carried out by restructuring the data array of the $^{60}\text{Co}$ source by changing the view# by a factor of 64. This correction number would be different if the total number of view used to scan the phantom changes. The $^{60}\text{Co}$ sinogram images shown in Figure 5-13 have been corrected for the view number; hence they appear identical to the $^{137}\text{Cs}$ sinograms. The monoenergetic images obtained from the projections data post view# correction (not shown here) have an identical one on one spatial orientation. The $^{60}\text{Co}$ corrected image matched the $^{137}\text{Cs}$ and the case I phantom from Figure 5-12. Unless specified all the images discussed in this section have been reconstructed based on the view# corrected data for the $^{60}\text{Co}$ source.

5.3.4 Phase holdup distribution: results and discussion

The sinograms from the two $\gamma$ energies for the two cases of the phantom have been processed using the ME-AM and PE-AM approaches discussed in Chapter 4. The image reconstruction was carried out using $80 \times 80$ pixel resolution. For easy comparison of the
results, the all holdup images have been plotted with a fixed color scale between 0 and 1. Since holdup is a fraction, it values should be in that range. The sectional holdup profiles through the images shown in this section have also plotted for a common scale.

The value of the attenuation coefficient of the materials that constitutes the phase is required to determine the hold by the ME-AM approach and the PE-Am approaches. Since the attenuation of water is well defined, and reported (Hubbell and Seltzer 1996) for a wide range of \( \gamma \) energies, the reported values were used. The NIST (Hubbell and Seltzer 1996) databases are able to provide the attenuation coefficient versus energy data for any material provided the composition of the material such that weight percentage of the different compounds or the elements that makeup the material is known. However, for the sold phase used in this study this data was not available from the supplier. A simple densitometry experiment was done to determine the attenuation coefficient of the solid phase. The 2 inch thick lead glass piece was placed in front of the detector to measure the unattenuated counts by the detector. The attenuation was then obtained using the Beer-Labert’s expression:

\[
\mu(E) = -\frac{1}{\rho l} \ln \left( \frac{I}{I_0} \right).
\]

The length \( l \) and density \( \rho \) are known, the counts values \( I_0 \) and \( I \) are measured in the absence and presence of the object respectively. This experiment was repeated a multiple time and the mean value of the attenuation was used for determining the phase holdup distribution.

**Monoenergetic approach:** The sinogram data was processed first to reconstruct the monoenergetic attenuation coefficient images using the monoenergetic alternating minimization. Equation (3-15) was iterated \( 10^4 \) times to ensure convergence. These images were then processed by the ME approach using (4-5) discussed in section 4.4 (Chapter 4) to
determine the holdup distribution images. The ME-AM holdup images of the gas, liquid and solid phases are shown in for the case I phantom in Figure 5-15. Similarly the phase holdup images of the case II phantom is shown in Figure 5-16.

Figure 5-15: Holdup distribution images for the case I of the phantom obtained using ME-AM. A: Gas holdup. B: Liquid holdup. C: Solid holdup.

Figure 5-16: Holdup distribution images of the case II phantom obtained using ME-AM. A: Gas holdup. B: Liquid holdup. C: Solid holdup.
**Polyenergetic approach:** The sonogram data was also processed using the PE-AM approach discussed in section 4.6 (Chapter 4). Equation (4 – 13) was iterated $5 \cdot 10^4$ times used to generate the holdup image for cases I and II of the phantom. Besides the nonnegativity constraint discussed in section 4.6 and additional constrain of keeping the maximum values holdup as unity was applied. Hence, at any $k$ if $\hat{e}^{(k+1)}_j(x) < 0$ it is overwritten as $\hat{e}^{(k+1)}_j(x) = 0$, and if $\hat{e}^{(k+1)}_j(x) > 1$ it is overwritten as $\hat{e}^{(k+1)}_j(x) = 1$. In this study the actual holdups of the phase are unity in the domains that they are present. This was not the case in the phantom studied in Chapter 4, section 4.3. It was found that in the range of iterations greater than $1 \cdot 10^4$ some the pixel would have holdup values $\hat{e}^{(k+1)}_j(x) > 1$ by a factor in the order of $10^{-3}$ or $10^{-2}$. Since this constraint is applied with in the iterative loop, it ensures that the solution is forced in the range of 0 to 1. The holdup images using the PE-AM approach have been shown in Figure 5-17 for case I of the phantom and in Figure 5-18 for case II.
Discussion on results: The holdup images using the ME-AM have yielded holdup distribution images that have a high error. The solid holdup is hardly captured in the case I of the phantom (Figure 5-15). In case II the solids domain is better captured, however the liquid domains C1 – C2 appear in the solids holdup image. The quality of the results obtained applying the ME-AM approach to the experimental projections data measured off the phantoms are similar to those observed in Chapter 4. Clearly the ME-AM doesn’t provide quantitative or qualitative information on the holdup distribution for three phase domains. The holdup distribution images obtained with the PE-AM algorithm are far superior for cases I and II of the phantom. The phase holdup images show a value of unity where the respective phase is present within the domain for both cases I and II of the phantom.
A closer look at the results is possible by examining a sectional holdup profile. The midsection profile of the phantom is shown in Figure 5-19 and Figure 5-20 for case I and case II of the phantom respectively. These profiles show the holdup values obtained along the vertical line along the diameter of the phantom. For both the cases the ME-AM based holdup profiles does not follow the ideal values. At a lot of instance they are out of the range of the scale. This indicates that for a lot of pixels the holdup values that are above unity or negative. This is certainly not possible as holdup values are volumetric fractions and have to have a value between zero and unity. The random fluctuations indicate that the small errors in the attenuation coefficient are amplified when holdup is calculated using (4 – 5). The holdup profiles of the PE-AM results follow the ideal values for the cases of the phantom. There are some fluctuation noticed in the holdup values but these are minimal. Since the holdup values have been constrained to be between less than or equal to unity, the holdup values appear flat when the profile reaches unity. Hence, the PE-AM results are far more meaningful and quantify the phase holdup distribution accurately.
Figure 5-19: Holdup profile along a vertical line (along the diameter) through the case I phantom.
Figure 5-20: Holdup profile along a vertical line (along the diameter) through the case II phantom.

For the data processing runs in the case the unity constrain on the holdup values was removed, as if \( \hat{\varepsilon}^{(k+1)}(x) > 1 \) is not overwritten as \( \hat{\varepsilon}^{(k+1)}(x) = 1 \). The results indicate that very few pixels have a holdup value greater than unity. The values were mainly in the range of 1.05 to 1.1 (holdup images not shown). Ghost rings are observed around the S1 – S3 and C1 – C2 subdomains in all the holdup images obtained using the ME – AM and PE – AM approaches. The rings are more pronounced in the ME-AM holdup images. This could be attributed to the blurring observed in the images reconstructed from the 1332 keV
photon data of $^{60}$Co source. An example is shown in the monoenergetic image of the phantom obtained with $^{60}$Co source in Figure 5-14A. The boundaries of the subdomains are blurred as compared to those in the image from the $^{137}$Cs source. The blurriness in the $^{60}$Co sinograms shown in Figure 5-13 A and C carries over to the reconstructed image in Figure 5-14A. This indicates that the back projector and other aspects related to execution of the reconstruction process have not introduced the blur.

In an experiment the projections data counts were collected by widening the energy windows (shown in Figure 5-10) such that lower end of the window is moved backwards to included: both the peaks, and both the peaks with some of the scatter. As the window got wider the sinograms of phantom showed less blurring. This $^{60}$Co sinogram with the widest window didn’t match the sharpness of the sinograms from the $^{137}$Cs source. Both the fan beams have the same physical dimensions, source size, dimensions of the SCD, the same type of detectors, and the same data acquisition hardware. This rules out the possibility of some experimental error. The reason for the appearance of the blurriness is not clear. When the energy window for the detector is expanded backwards to include count with lower energy seems to suggest that this could be a problem related to either the higher energy of the photon from the $^{60}$Co source.

The incident photons counts obtained from the 50 mCi $^{60}$Co are lower than those from the 250 mCi $^{137}$Cs source. When the energy window is widened the incident photon counts increase. The incident photon counts from the 1332 keV peak of the $^{60}$Co source are less by a factor of 3 when compared to the 661 keV peak of the $^{137}$Cs source. The sinogram of the $^{60}$Co is still blur when the energy window is expanded such that it matches incident photon counts from the the $^{137}$Cs source. This rules out poor count rate as a source of the blur. Clearly, there is an indication that this problem is related to the energy of the photons.
as the expanded window still has count of high energy than the $^{137}\text{Cs}$ source. Further experiments are required to determine the reasons and limitations, if any, of the NaI crystal based detectors used in the fan beam. The blurring effect has not drastically effected the holdup distribution results with the PE-AM algorithm. If dynamic flow is imaged the possibilities of a sharp edge in the flow domain is further reduced as this CT measures time averaged holdup profile of the flow.

5.4 Remarks

The DE-DSCT developed has a number of practical advantages on the earlier single source computed tomography system at CREL. It could be used study distribution in two phase systems with redundancy in the data as two sources are used. As both the sources emit high energy $\gamma$ photons it can scan multiphase flow systems with large domain size or high attenuating material in the flow. The design includes separate fan beams on the DE – DSCT for the two energies which helps minimizes pollution of the measured counts data with Compton scatter counts from the higher energy $\gamma$ photo peak. The data acquisition hardware has the ability to set a lower and higher bin specification to form a window such that only the counts of the desired energy are included in the data. This ability not only helps avoid measuring the Compton scatter from the peak of interest but also allows for section of specific energy of the photo peak if the source has multiple $\gamma$ energy peaks. In this case it is possible to separately measure the counts of the 1332 keV and 1173 keV photo peaks from the $^{60}\text{Co}$ radio isotope. It is possible to set 8 such windows using the DE-DSCT data acquisition hardware. In principle a single fan beam can accommodate a $\gamma$ emitting isotope with 8 distinct photo peaks, and be capable of minimizing Compton scatter in measurements of the counts from the peaks.
The dual energy computed tomography systems reported in literature (discussed earlier in Chapter 2) are not able to image phase holdup distribution in large domains with three phase flow. Researchers typically use of the post processing ME approach using forms similar to (4-5) from section 4.4 in Chapter 4. Since equation (4-5) is ill conditioned at higher energies of the $\gamma$ photons low energy $\gamma$ photons have to be used. This invariably means it can only be applied to small domains. The maximum domain size of 3 inches has been reported by Froystein et al (Froystein et al. 2005) with the use of special low attenuating polymer for constructing wall of the flow domain. Bukur et al (Bukur et al. 1996) reported a densitometry study on an 8 inch diameter slurry bubble column (SBC) to determine the radial phase holdup profiles using the $^{60}$Co and $^{137}$Cs sources. The authors reported failure of the experiments as the high error in the results due to ill conditioned system of equations rendering them meaningless.

The DE-DSCT was successfully validated with a three phase phantom. The holdup distribution results shown in this chapter provides experimental evidence confirming the possibility of imaging phase holdup distribution of three phase system using the $^{60}$Co and $^{137}$Cs isotopes. The main reason for the success is attributed to the mathematical approach of the PE-AM algorithm (discussed in Chapter 4, section 4.6) for processing the transmission data from the isotopes. The results indicate the desired level of accuracy expected from a tomography technique that could be used as research tool for understand three phase flow in various multiphase systems.
Chapter 6

Effect of Sparger Design on the Hydrodynamics of a Biogas – Lift Anaerobic Bioreactor

6.1 Introduction

From the literature review discussed in Chapter 2 and the review by Vesvikar (Vesvikar 2006) it is evident that biogas recirculated reactors are a favorable mode for mixing in anaerobic bioreactors. Since the gas is be introduced into the system such that it induces mixing, the gas holdup distribution which in turn effects liquid circulation are important aspects that need to be studied. The sparger design and has a direct bearing on the gas holdup, especially in systems wit a low length to diameter ratios, the study of its effect on mixing would aid better design of anaerobic bioreactors. In this chapter the effect of sparger design on the hydrodynamics of a labs scale surrogate anaerobic bioreactor has been studied.

For anaerobic degradation to be commercially viable the process should generate net energy while disposing of waste. Hence, the amount of process energy that can be invested is limited by the quantity of energy generated from the methane produced. Although the other byproducts such as the solid sludge, has great potential as soil conditioner and fertilizer, its commercial value is limited. Typically, the modes of mixing in anaerobic bioreactors can be broadly classified as impeller based agitation, liquid jets or slurry recirculation, and gas recirculation. High mixing rates may yield better performance in biogas generation; however
when the process energy requirement is weighed against the energy from the biogas generated, a process with high mixing rates becomes economically unviable. Also, very high mixing rates may cause destruction of the microbial population (Stroot et al. 2001; Whitmore et al. 1987), which would lead to the failure of the bioreactor. It is for this reason that impeller based reactors and circulating liquid jets are not popular.

Biogas recirculation bioreactors are a more appealing option as they have no moving parts and their energy requirements are minimal. In these reactors the biogas generated is recirculated with the aid of blowers. It must be noted that the gas in anaerobic processes is used purely to induce mixing and does not consists of any species that participates in reaction or in the cell growth process. In contrast, gas (air) used in aerobic process has a species (oxygen) that participates in the cell growth processes. Therefore, requirements for high gas holdup or a high gas-liquid interfacial area, to facilitate mass transfer of any species from the gas phase to the liquid phase do not exist in the anaerobic bioreactors. This basic reality makes a significant impact on the operating conditions and design of gas mixed bioreactors for anaerobic digestion applications. According to an estimate by Chisti (Chisti 1998), aerobic reactors that are gas (air) agitated require 3000-2000 W/m³ to meet the dissolved oxygen and mixing requirements for systems that produce primary metabolites from microorganisms or that are used for effluent treatment. Systems that grow cells normally require energy input in the range 100 W/m³. For gas mixed anaerobic systems the EPA (EPA 1979) recommends an energy input range of 5-8 W/m³. This shows that there a few orders of magnitude difference between the energy requirements of gas mixed aerobic and anaerobic bioreactors.
In this work the effect of a single orifice sparger (SOS) system, also called an ejector, on mixing and hydrodynamics has been compared with that of a multi orifice ring sparger (MORS). The gas phase distribution, the liquid velocity profile, and the liquid flow pattern visualization studies have been conducted for the same superficial gas velocities in both the systems. Such comparison will indicate the impact of the degree of uniformity of gas holdup in the draft tube on the formation of the poorly mixed zones in the system. These studies have been done with the synergistic use of a single source $\gamma$ ray Computer Tomography (CT) and Computer Automated Radioactive Particle Tracking (CARPT). CARPT and CT are not hindered by typical opacity of reactor walls and its contents. CARPT enables mapping the flow field of a particular phase (solid or liquid) in a given system.

6.2 Materials and methods

6.2.1 Details of the surrogate anaerobic bioreactor

The experiments were carried out in a 6 in. ($15.24 \times 10^{-2}$ m) diameter bioreactor with a conical bottom and a draft tube (Fig.1 and Fig.2). The draft tube diameter was selected so that the ratio of the draft tube’s internal diameter ($D_{di}$) to that of the internal diameter of the reactor ($D_r$) was 0.25. Various researchers in the past have worked with air lift reactors with various draft tube diameter to reactor diameter ratios (Kojima et al. 1999; Pironti et al. 1995). Their observation was that lower draft tube diameters to reactor diameter ratios tend to give a higher liquid velocity in the draft tube. In a study by Karim et al. (Karim et al. 2004) in an 8 in. ($20.32 \times 10^{-2}$ m) diameter gas recirculation bioreactor, the draft tube diameter to reactor diameter ratio was 0.2. Walker Process Inc., (Aurora, IL 60506-285), one of the commercial manufacturer of biogas recirculation anaerobic bioreactors for municipal wastes, makes
bioreactors with ejector tubes (similar to the SOS system) with a draft tube diameter to reactor diameter ratio of 0.07 to 0.08. Since the aim was to study the effect of sparger design, the draft tube to reactor diameter ratio was chosen in the range of that studied by previous researchers. The draft tube was spaced equidistant from the top and the bottom of the active region of the bioreactor, along its vertical axis at the center. The effect of the conical bottom has been studied in a flow pattern visualization study, using computational fluid dynamics, by Vesvikar and Al-Dahhan (Vesvikar and Muthanna 2005) for surrogate gas recirculation anaerobic bioreactors. One of the conclusions reached was that a 25° sloping angle from the horizontal gave the better performance for a given gas flow rate. Hence a conical bottom with a sloping angle of 25°from the horizontal plane was used for this study.

The single orifice sparger (SOS), as shown in Figure 6-1, consisted of a long 0.5x10^{-2} m i.d. steel tube which opens into the reactor at the bottom of the draft tube. The steel tube was fixed to the upper lid of the reactor. There were four flat tie rods at the upper and the lower ends of the draft tube. The rods are welded on one end to the steel tube and on the other end to the draft tube. This arrangement supported the draft tube and kept it in the desired position during operation. The gas was introduced through this tube from the top of the reactor, and it entered the reactor at the lower end of the draft tube.
Single orifice sparger (SOS)

Figure 6-1: Cross section of the surrogate anaerobic bioreactor used with a single orifice sparger (SOS) (Varma and Al-Dahhan 2007).

Multi – orifice ring sparger (MORS)

The multi-orifice ring sparger (MORS), as shown in Figure 6-2, consisted of a tubular ring supplied with gas by three central tubular arms. The diameter of the orifices on the ring was $0.1021 \times 10^{-2}$ m. Twenty-four equally spaced orifices were drilled in the ring
(Figure 6-3). This way the open area (all orifices included) of the MORS was same as that of the SOS. The gas was introduced from the bottom of the bioreactor through a steel tube into the center of the MORS where the three cross arms meet. The MORS is set at 0.048 m from the bottom of the bioreactor. The orifices face downwards at an angle of 30° from the vertical (Fig. 2(b)), and face alternately inwards and outwards in pairs. The orifices open downwards to minimize the possibility of being choked or clogged with solid debris that circulates in the bioreactor slurry. Since the anaerobic bioreactor involves a gas-liquid-solid system like slurry bubble columns with fine catalyst particles, the precedent of downward facing pores in the sparger set by George et. al. (George et al. 2001) and Ong (Ong 2003) was followed in this case. The orifices of the MORS opened into the system at a height of 0.042 m from the bottom of the reactor. Hence, the gas in the MORS system also entered at the same axial position as in the SOS system.

The draft tube support system consisted of four flat tie rods that connect to a flat circular strip which had an internal diameter that matches the draft tube’s outer diameter. This strip was attached with fasteners (not shown in Figure 6-2) to the bottom of the draft tube. The four tie rods converged at the center, where they were welded to the wall of a steel tube. This steel tube was attached to the bottom of the reactor (at the center of the conical section) with fittings. The steel tube attached to the center of the MORS was of smaller diameter than the steel tube connected to the tie rods of the draft tube support system and was able to fit into it. This assembly was clamped to the outer tube. The gas line was attached to the tube that was connected to the MORS.
Figure 6-2: Cross section of the surrogate anaerobic bioreactor used a multi orifice ring sparger (MORS) (Varma and Al-Dahhan 2007).

Figure 6-3: Details of the multi orifice ring sparger (MORS) used (Varma and Al-Dahhan 2007).
6.2.2 Experimental conditions

This study used bovine manure from the University of Tennessee (UT) dairy farm at Oak Ridge, TN. The waste was pre-treated before use by wet screening through a 2 x 10^{-3} m sieve, followed by dilution. For this slurry the total solids (TS) level was set to 50 kg/m^3, and the volatile suspended solids (VSS) concentration was found to be 3.45 kg/m^3. A total volume of 4.2 x 10^{-3} m^3 for each reactor was used in the study. The gas flow rates ranged from 0.017 x 10^{-3} m^3/s to 0.083 x 10^{-3} m^3/s. Details of flow rates and the superficial gas velocity based on the draft tube diameter are given in Table 6-1. The gas flow rate was regulated with the aid of a rota-meter attached to the air line.

Air was used in this study to substitute the biogas (a mixture of CH_4 and CO_2). Karim et al. (Karim et al. 2005b) conducted performance studies with four anaerobic bioreactors mixed by different modes processing bovine manure from the same dairy farm. One of these included a configuration similar to the SOS system. The authors reported a composition ratio of CH_4:CO_2 ranging from 56-64%: 44-36% in the biogas produced. In another performance study by Borole et. al. (2006) involving a gas mixed reactor with bovine manure, the ratio of CH_4:CO_2 was found to be 60:40. The molecular weight of biogas with these compositions is in the range 28.32-26.08 kg/kmol. Based on these findings air was used to substitute the biogas. Another reason for using air is that the CAPRT procedure used for liquid velocity measurement (described later) involves a calibration step that involves the insertion of a rod into the system. A hermetic system would not allow access to this calibration rod and hence would complicate the CARPT calibration process. The study by Karim et al. (Karim et al. 2005b) also indicates that the biogas generated in 24 hrs (once the reactor operation has reached steady state) is only about 3% of the total volume of the
Gas recirculated through the sparger at a rate of $0.017 \times 10^{-3} \text{ m}^3/\text{s}$. Hence it can be concluded that the hydrodynamic effects due to the gas in the system are mainly caused by the recirculated biogas, and the contribution of the biogas generated by waste degradation is minimal.

The process power input into the reactor, via gas, was calculated using the expression (6-1) developed by Casey (Casey 1986)

$$\frac{P}{V} = \frac{G_r P_2}{(\lambda - 1)/\lambda} \left[ \left( \frac{P_1}{P_2} \right)^{(\lambda-1)/\lambda} - 1 \right]. \quad (6-1)$$

$P$ is power, $V$ is the active volume of the reactor, $G_r$ is the biogas recirculation rate, $P_2$ is the pressure in the head space, $P_1$ is the pressure at the point where gas in introduced ($P_2$ static ahead of slurry) and $\lambda = 1.03$ (Casey 1986). The range of power input (based on gas flow rate) used in this study is within the range of 5-8 W/m$^3$ (summarized in Table 6-1) suggested by the EPA (EPA 1979) for such systems. Therefore it can be safely said that the surrogate anaerobic bioreactor system used in this study very closely represents a lab scale anaerobic bioreactor. Since this (6-1) doesn’t have terms that are dependant on the open area or other design parameters of the sparger, it is assumed that for a given gas flow rate the same amount of energy is introduced into both the systems. Hence the conclusions drawn in this study represent an actual system with a great degree of accuracy.

Table 6-1: Flow rates conditions used and Error in calibration process for anaerobic bioreactor with SOS and MORS systems
<table>
<thead>
<tr>
<th>Gas flow rate ( Q_g \times 10^3 ) ( (m^3/s) )</th>
<th>Superficial gas velocity based on draft tube diameter ( \times 10^3 ) ( (m/s) )</th>
<th>Superficial gas velocity based on reactor diameter ( \times 10^3 ) ( (m/s) )</th>
<th>Power Input ( (W/m^3) )</th>
<th>Average Calibration Error for reactor with MORS ( \times 10^3 ) (Standard Deviation ( \times 10^3 ))</th>
<th>Average Calibration Error for reactor with SOS ( \times 10^3 ) (Standard Deviation ( \times 10^3 ))</th>
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<td>1.47</td>
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<td>8.71</td>
<td>0.26 (0.14)</td>
<td>0.37 (0.21)</td>
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<td>26.15</td>
<td>0.37 (0.24)</td>
<td>0.38 (0.19)</td>
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<td>7.35</td>
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</tr>
</tbody>
</table>

### 6.2.3 Overview of computer automated radioactive particle tracking (CARPT) technique and experimental procedure

CARPT is a powerful method employed for measuring the flow field, instantaneous time averaged velocities and turbulent parameters, dead zones, residence time distribution, and other parameters of a particular phase in a given system. It tracks the motion of a radioactive particle that represents an element, or small packet, of a phase in a system. It is usually used to track liquid or solid phases (Devanathan 1991). For tracking liquids, the density of the radioactive particle (usually a few hundred microns in size) is made to match the density of the liquid by encapsulation in a polypropylene (PP) ball. Such a particle is tracked with the aid of an array of NaI scintillation detectors placed at strategic locations around the system. A typical CARPT study involves the following sequence of steps: preparation of radioactive particle, in situ calibration (at experimental conditions), experimental runs, and processing of the data generated from the preceding two steps (Roy 2000). These aspects have been discussed in brief here to maintain continuity, for further
details the reader is advised to refer to the works of Karim et. al. (Karim et al. 2004), Devanathan (Devanathan 1991), and Degaleesan (Degaleesan 1997).

**Particle Preparation**

In this study the liquid phase was tracked. A \(^{46}\)Sc radioactive particle of 300 \(\mu\text{Ci}\) strength and 150 micron diameter was encapsulated in a PP ball of 0.001 m diameter with a calculated air gap. The density of the composite particle was made to match that of water. This was done by determining the density of the composite particle based on its terminal settling velocity and Stoke’s law:

\[
U_t = \frac{gD_p (\rho_s - \rho_w)}{18 \mu_w}.
\]  
(6-2)

To determine the settling velocity the particle was released in a 1 m long acrylic tube filled with water. The terminal settling velocity was calculated based on the time taken to settle in the last 0.5 m of the tube. Then using (6-2), its density was calculated. If the particle was light, it was coated with commercial spray paint to make it heavy. If it was heavy, it was made lighter by coating it with a commercial epoxy glue spray with a density less than that of water. After repeated adjustments the composite particle used for the experiments had a density of 1.0016 kg/m\(^3\) (average absolute error = 0.0016, standard deviation = 0.0012: based on five terminal settling velocity measurements).

**Calibration**

The gamma ray photon counts received by the NaI scintillation detectors are a function of the distance of the radioactive particle from the detector, and of the attenuation
of the photons due to the material between the detector and particle. Hence the calibration process is carried out in situ, i.e. the reactor was operating at the given conditions and configuration for which the experiment was to be carried. Thus, the number of counts received during calibration matches the counts received if the free particle were to be at the same location in the tracking experiment (described later). For a given signal strength (or the number of counts) detected for a particular detector, the particle could be anywhere along the surface of a hypothetical sphere with a radius represented by the distance the corresponding counts or signal strength refer to. Hence a minimum of three detectors are required to pinpoint the location of a particle. A total of sixteen detectors were used in this system, as indicated in Figure 6-4.

Figure 6-4: 16 NaI scintillation detectors are located as indicated. The bioreactor is placed in the center
The composite particle is placed in a small teflon capsule attached to the tip of a small diameter (3/8 in.) steel rod. The steel rod is attached to an automated CARPT calibration device (Luo 2005b). This device is capable of being programmed to move the rod, and the particle on its tip, to multiple known locations within the system. The calibration rod is introduced into the system from the upper side of the reactor by opening the lid or top cover. The counts received by all the detectors used in the system, for each location of the particle, are recorded via a data acquisition system. The counts were acquired under the $^{46}$Sc photo peak (the $\gamma$ photo peaks for $^{46}$Sc are at 889 and 1120 KeV). Multiple samples (180 samples) are collected for each location at a sampling rate of 50 Hz, i.e., the counts received by the detectors are recorded every 0.02 seconds. For the MORS system the particle was placed at 1723 positions within the reactor, at increments of $\Delta Z = 1.27$ cm along the axial direction, $\Delta \theta = 20^\circ$ azimuthally, and $\Delta r = 1.27$ cm radially. The area underneath the draft tube was inaccessible to the calibration rod and hence was not covered. Similarly, 1392 points were covered in the SOS system. The numbers vary for the MORS and SOS as difference in internals affect accessibility of the calibration rod in the system. The mean of the counts data for each location was used to calibrate the system. A $\beta$ spline fit technique developed by Devanathan (Devanathan 1991) was used to fit the data. In this manner a distance-count map was generated for the reactor. The details of the procedure and the electronic hardware used are described elsewhere (Degaleesan 1997; Devanathan 1991; Rados 2003b).

To test for error in the calibration, the counts data for the known positions of the particle was split into two groups, which constitute 80% and 20% of the complete data set. The 80% data set was used to calibrate the system by developing a distance-counts map. The 20% data set was traced based on this distance count map. The traced positions were
compared to the positions (distance) at which these data points (the 20% set) were taken. Error was determined by taking the mean of the differences in the position. The results, summarized in columns 5 and 6 in Table 6-1 show that the average error was 0.3 cm.

**Particle Tracking**

Once the calibration was done, the particle was removed from the calibration rod and is introduced in the system. The particle was allowed to move freely within the system, unhindered by external influence. While the particle was moving in the reactor operating at the set conditions, the counts received by all the detectors were recorded at the same sampling rate as it was calibrated (in this case, 50Hz). This tracking process was carried out for 18 hours for all the conditions described in Table 6-1.

**Data processing**

To reconstruct the position of particle, the distance-count map generated by the calibration process was used to process the counts data obtained during tracking. Each location of the particle was at a time interval of 0.02 seconds; hence the instantaneous position (i.e., Lagrangian trajectory) of the particle was obtained. The process involves the application of a weighted least-squares algorithm, a wavelet based position filtering, and by a particle position reconstruction algorithm described elsewhere (Degaleesan 1997; Devanathan 1991; Rados 2003b), to determine the best estimate of the particle position. Time differencing between two positions yielded instantaneous velocities, which were averaged at each spatial location over the whole time span of the experiment to yield the ensemble average velocity flow map of the system. To get Eulerian information, the volume of the reactor was divided into 28,160 cells (20 divisions in r, 32 in \( \theta \), and 44 in the z direction, respectively). The estimated instantaneous velocity information was then assigned
to the compartment falling at the midpoint of two successive particle positions. The difference between the instantaneous and the average velocity for each cell yielded the fluctuating component of the velocity as a time series, which could also be used to calculate the turbulent parameters. The average liquid velocity data was used for comparison in this study.

6.2.4 Overview of computer tomography (CT) technique and experimental procedure

The details of the mechanical hardware and the data acquisition system for the single source $\gamma$ ray CT unit used in this study has been described at length by Roy (Roy 2006a). This CT used the $^{137}$Cs emitting $\gamma$ photons at 661 keV as the $\gamma$ ray source. The AM algorithm proposed O'Sullivan and Benac (O'Sullivan and Benac 2007) and implemented by (Varma et al. 2008) was used for image reconstruction. Single source CT can be used to compute the individual phase holdup of only a two phase system. In this study the liquid and solids were considered as a single slurry phase, as the solids concentration was only 5% (w/v basis). The bio-solids have attenuation close to that of water at 661 keV and at 5% the solids contribution to attenuation is minimal. The attenuation coefficient is a function of the density of the material and the atomic number of the elements that constitute the material. The biosolids have a density that is marginally greater than water and don’t have elements with greater atomic number than the elements that constitute water. Simple densitometry measurements were made to determine the difference between the attenuation of water and slurry in the reactor. The results indicate no significant difference. Also, pretreatment avoided large debris and sand particles in the system that would cause a higher attenuation than water. Based on this a pseudo two phase assumption was made in this study.

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The procedure described in Chapter 3 section 3.2.1 was followed to determine the attenuation coefficient images of the flow. Phase holdup distribution for the gas phase was determined using (3-29) and for the liquid was determined using (3-30) from section 3.3 of Chapter 3. The equations have not been repeated here. The CT Scans of the systems with SOS and MORS were carried out at level 1 (5x10^{-2} m from the base of the reactor) and at level 2 (15x10^{-2} m from the base of the reactor) as indicated in Figure 6-1 and Figure 6-2 respectively.

6.3 Results and discussion

6.3.1 Computed tomography

Effect of gas flow rate and sparger configuration on gas holdup distribution.

Figure 6-5: Three dimensional representation of cross sectional time averaged gas holdup distribution in the draft tube region for A: system with SOS and B: system with MORS at level 1 (5x10^{-2} m) and level 2 (15x10^{-2} m) at Q_g = 0.05 x10^{-3} m^3/s (superficial draft tube gas velocity =4.41x10^{-2} m/s). Color bar represents the holdup value (Varma and Al-Dahhan 2007).
Figure 6-5 shows a three dimensional perspective of the tomograms showing the
time averaged gas holdup distribution of the system with the SOS and the MORS, at levels 1
and 2 in the draft tube region with a gas flow rate of $0.05 \times 10^{-3} \text{ m}^3/\text{s}$. These images have
been reconstructed on the basis of (3-29). The draft tube region alone has been shown here
as the tomograms generated for the entire cross section (not shown here) show that gas
holdup in the region outside the draft tube in the reactor is negligible. This can be seen in
Figure 6-7 which shows the radial gas holdup profile (discussed later).

The gas spreads within the draft tube as it is released and rises in the draft tube for
both the systems. The MORS system has more gas distributed in the draft tube than does
the SOS. In Figure 6-5A there is a local spot of high gas holdup visible at the center within
the SOS system at level 2 ($15 \times 10^{-2} \text{ m from the base of the reactor}$). This represents the gas
inside SOS injector tube that extends from the top of the reactor to the bottom region of the
draft tube (Figure 6-1). There is also a concentration of the gas phase near the center of the
draft tube surrounding the SOS injection tube. This shows that there is certain degree of
channeling of the gas within the central region of the draft tube in the reactor with the SOS
system, as it rises within the draft tube after it is introduced at the bottom of the draft tube.

The gas holdup distribution in the draft tube of the MORS system is quite different.
The gas holdup is confined to circles over the ring sparger at level 1 ($0.05 \text{ m from the base}$),
as level 1 is located very close to the MORS. The gas then distributes itself over the region of
the draft tube for the system with MORS as it rises, as seen in at level 2 in Figure 6-5 B. For
both the SOS and the MORS, the gas flow rates were maintained at $0.05 \times 10^{-3} \text{ m}^3/\text{s}$ in Figure
6-5, hence a better gas holdup distribution is observed in the draft tube region of the
bioreactor with MORS.
Figure 6-6: Tomograms of system with MORS at level 1 with gas flow rates (a) 0.017x10^-3 m^3/s, (b) 0.05x10^-3 m^3/s and (c) 0.083x10^-3 m^3/s (Varma and Al-Dahhan 2007).

The presence of the local circular regions of gas holdup above the sparger also indicates that for the given flow rate not all the pores in the MORS open up. At a higher gas flow rate (0.082 x10^-3 m^3/s), more open pores are observed in the tomogram as shown in Figure 6-6. There is insufficient pressure drop across the sparger at lower flow rates to open all the pores in the sparger. The gas holdup tomograms do not have resolution high enough to judge the exact number of open pores in the MORS. It must be noted that even with a
few pores open at low gas flow rates, the MORS still gives a better gas holdup distribution than the SOS.

An alternative, to ensure all pores open, could be to reduce the diameter of the pores further (and thereby reducing the open area) without compromising the number of pores. However, given the nature of the bovine manure slurry and its inconsistency in terms of the characteristic of the solids present in it, the chances of pore clogging would be high during operation for very small pores. The other alternative could be to reduce the number of pores in the sparger such that the minimal gas flow rate covered in the study would open all the pores. This in turn could reduce the number of pores drastically (depending on the gas flow rate it would be designed for). When this design (with reduced number of pores) is used for higher flow rates the holdup distribution would be affected as the gas would be confined to a limited number of pores versus a scenario where it could have been introduced through a higher number of pores, thereby giving better gas holdup distribution. For this study, having a different sparger for each of the gas flow rates with specific number of pores (such that all are open during operation) would have been impractical.

Figure 6-7 depicts the azimuthally averaged mean gas holdup profiles in the reactor at levels 1 and 2 respectively, where the gas holdup is plotted versus radius. The values for all the flow conditions covered in Table 6-1 are shown for both SOS and MORS systems.
It is clearly visible that at level 1 (Figure 6-7 A) for the case of the SOS there are certain local spots where the gas holdup is higher than that of the MORS for a given gas flow rate. The local peak (at a radius of 1x10^{-2} m) in the gas holdup profiles of the MORS is due to the fact that the location of level 1 is just above the MORS; hence the gas coming out the orifice is responsible for this peak. At level 2 (Figure 6-7 B), the gas holdup distribution is more radially uniform in the MORS system. In contrast, all the profiles for the SOS system show a distinct peak near the center of the reactor.

A better understanding can be obtained by observing the mean of the gas holdup profile. Figure 6-8 shows the mean gas holdup in the draft tube versus the superficial gas
flow velocity. The superficial gas velocity is based on the gas flow rate and the cross-
sectional area of the draft tube. The mean gas holdup is calculated using:

\[
\bar{\varepsilon}_g = \frac{\sum_{x=1}^{n_{\text{draft}}} g(x)}{n_{\text{draft}}}. \tag{6-3}
\]

Here \( x \) represents the pixel index, and \( n_{\text{draft}} \) the number of pixels in the draft tube
region. Since the area each pixel represented in the domain is equal, \( \bar{\varepsilon}_g \) in (6-3) also
represents the area weighted average gas holdup.

Figure 6-8: Mean gas holdup in the draft tube region of the SOS and MORS systems at
levels 1 and 2 (Varma and Al-Dahhan 2007).

Figure 6-8 clearly confirms that the mean gas holdup is higher for the MORS system
at both levels 1 and 2. As gas flow rate increases, the gas holdup increases. This rise in gas
holdup with gas velocity is more pronounced in the case of the MORS system. The
superficial velocities’ effect on gas holdup is well known. However, what is highlighted here is that the effect sparger design and its impact on gas holdup distribution. For a fixed superficial gas velocity the system with MORS gives a higher (spatially) average gas holdup in the draft tube. The impact of this enhancement in gas holdup distribution on the liquid velocity and poorly mixed zones outside the draft tube will be discussed in the CARPT results section.

**Characterization of the uniformity of gas holdup distribution**

It is important to quantify the uniformity of the cross-sectional gas holdup distribution in order to properly compare the effects of the design and operation parameters of the bioreactor. In this section the different approaches have been discussed with their merits and demerits.

Maldistribution in flows, particularly those that contain liquids, has been characterized at length in the literature. Some of these approaches divide the domain of flow into sub domains of a given size. Then statistical tools are used to compare the holdup in this sub domain with the global mean (i.e. mean of the entire domain). Mercandelli et al. (Marcandelli et al. 2000) calculated the maldistribution factor for liquid flow in gas-liquid packed columns using (6-4). The flow domain was divided into nine sectors (denoted by N)

\[
M_f = \sqrt{\frac{1}{N(N-1)} \sum_{n=1}^{N} \left( \frac{e_{i,n} - \overline{e_i}}{\overline{e_i}} \right)^2},
\]

(6-4)

Here \( M_f \) is the maldistribution factor, \( \overline{e_i} \) is the mean liquid holdup (based on the entire domain) and \( e_{i,n} \) is the sub domain gas holdup. Roy (Roy 2006b) has used a T-Test
based method on liquid saturation data in packed monolith reactors obtained from CT. In this approach the main domain was divided into several sub domains of 6 pixels each. Each of these sub domains was compared with a hypothetical sub domain of the same size (6 pixels) that has the mean holdup values calculated based on the entire domain (global mean), with the aid of the T-test with a 95% confidence interval. If the sub domain ‘passed’ the t-test, the null hypothesis was satisfied, the two groups were statistically the same, and the value 1 was assigned to that sub domain group. If it failed then a 0 was assigned. Finally a uniformity factor was determined as a percentage of the sum of domains labeled with 1. The sub domain T-test has the right conceptual fundamentals and is able to give a quantitative value to degree of uniformity in terms of a percentage, which is convenient for the purpose of comparison.

However, domain based methods in general have the two drawbacks. The first drawback is the strategy one uses for choosing the size and orientation of the sub domain. Experimentalists who use collectors to make liquid distribution measurements in structured columns typically create sub domains in the collectors in the order of 16-25 in number, and hence find it convenient to use a sub domain oriented strategy. Since the orientation of the sub domain remains constant in the set up, liquid flow distribution with different packing materials and operational conditions can be compared. However if one were to vary the sub domain size, the results differ. The second drawback is the problem that could evolve in situations where the holdup of the particular phase of interest is small and confined to a small region. While computing the mean holdup of the entire domain in order to carry out the T-test, the weighted contribution of the holdup to all the pixels, from the few that have the phase in them, may be very small. When the T-Test is done there wouldn’t be any
statically significant difference in the mean of the sub domain and the global mean. Hence the analysis would indicate that the system is very well distributed, which would be counter intuitive to the visual observation in the tomogram.

Figure 6-9: Plot showing dependency of distribution factor on number of pixel per block used for computing distribution factor based on T-Test. Gas holdup distribution data from the scan at level 2 for the MORS system with \( Q_g = 0.05 \times 10^{-3} \text{ m}^3/\text{s} \) was used (Varma and Al-Dahhan 2007).

To take a better look at the effect of the domain size on the gas holdup distribution, the MORS system data at \( 0.05 \times 10^{-3} \text{ m}^3/\text{s} \) gas flow rate at level 2 was subjected to the T-test based uniformity factor analysis using the Roy (Roy 2006b) approach. There are 436 pixels in the domain representing the cross-section of the draft tube. Sub domains with pixels blocks varying from 5 to 50 were used yielding results shown in Figure 6-9. Clearly the distribution factor varies as the size of the domain changes. There is no asymptote visible in Figure 6-9, even where small numbers of pixels are used for a sub domain, which would make the number of sub domains large. However, if the holdup distribution analysis is performed for different flow conditions in a given experimental setup then one fixed sub domain size may be considered as a basis for comparison. This type of analysis would not inspire confidence in characterizing holdup distribution in a system for universal comparison.
The T-test based analysis was carried out for this study, using a sub domain size of 6 pixels. The results are summarized here. For $V_g = 1.46 \times 10^{-2}$ m/s ($Q_g = 0.017 \times 10^{-3}$ m$^3$/s), at both level 1 and level 2 the uniformity factor for the MORS system is a little more than double as that of the SOS (Level 1: MORS = 61.0 %, SOS = 27.8 %; Level 2: MORS = 53.2 %, SOS = 30.5 %) system. There is a similar trend for $V_g = 4.41 \times 10^{-2}$ m/s ($Q_g = 0.05 \times 10^{-3}$ m$^3$/s) at both levels (Level 1: MORS = 42.5 %, SOS = 21.1 %; Level 2: MORS = 51.0 %, SOS = 33.3 %). However, when compared to the previous flow rate, the uniformity factor values appear to be lowered. Finally for $V_g = 7.35 \times 10^{-2}$ m/s ($Q_g = 0.083 \times 10^{-3}$ m$^3$/s), the values for distribution at levels 1 and 2 are almost comparable for the MORS and SOS systems (Level 1: MORS = 36.2 %, SOS = 23.2 %; Level 2: MORS = 21.0 %, SOS = 21.3 %). These observations are contrary to tomograms for these levels (not shown) and the azimuthally averaged gas holdup profiles shown in Figure 6-7.

A simpler statistical method is developed in this work where the variance of the holdup distribution of the entire domain is calculated based on the gas holdup in each pixel in the domain. The variance is normalized by the mean gas holdup for that domain. This non-dimensional variance (Equation 11) can be called a mal-distribution factor. This equation bears resemblance to the ones used by Marcandelli (Marcandelli et al. 2000) and Jiang (Jiang 2000) to characterize mal-distribution

$$\sigma = N_{Mal} = \left( \frac{1}{n} \sum_{x=1}^{g} \left( \frac{\epsilon_x(x) - \bar{\epsilon}_g}{\bar{\epsilon}_g} \right)^2 \right).$$  (6-5)

If the variance of the holdup data is zero, this would indicate that that holdup profile is absolutely uniform and even (mal-distribution is zero). If non-dimensional variance tends
towards unity, it is mal-distributed. One must be aware that this value could be greater than unity in severely mal-distributed cases.

It is therefore better to call this ratio a ‘mal-distribution number ($N_{Mal}$)’, just like any non-dimensional number, as factors or coefficients tend to be between zero and unity. This method is more simple and fundamental and not dependent on the size or geometrical orientation of the sub domains involved. Also, as the number of pixels ($n$ in (6-5)) increases, by statistical principles the accuracy of $N_{Mal}$ will also increase.

![Figure 6-10: Normalized standard deviation or maldistribution number ($N_{Mal}$) based comparison for gas phase distribution in the draft tube region for a SOS system and MORS system (Varma and Al-Dahhan 2007).](image)

$N_{Mal}$ values based on (6-5) are shown in Figure 6-10. It should be noted that the smaller the $N_{Mal}$ value, the better the gas holdup distribution. Clearly, for $V_g = 1.46 \times 10^{-2}$ m/s ($Q_g = 0.083 \times 10^{-3}$ m$^3$/s) there is difference in $N_{Mal}$ by a factor of 2 (over 100% difference) at Level 2 for the SOS and MORS systems. This difference is also seen at $V_g = 4.41 \times 10^{-2}$ m/s ($Q_g = 0.05 \times 10^{-3}$ m$^3$/s). At higher $V_g$ values, this difference narrows a bit, but
still differs by a factor of 70-80% between the SOS and MORS systems. All the \( N_{Mal} \) values for SOS are above unity, this indicates severe maldistribution of the gas in the draft tube region for the system with SOS. Hence it can be concluded that the MORS, in spite of partially opened pores (Figure 6-6), gives a better gas holdup distribution than the SOS for a given gas flow rate.

6.3.2 Results from computer automated particle tracking (CARPT)

Liquid velocity and flow pattern in digester with MORS and SOS

Figure 6-11: Time averaged velocity vector plots for digester with 5% (TS) solid loading slurry at for gas flow rate \( Q_g = 0.05 \times 10^{-3} \text{ m}^3/\text{s} \) (\( V_g = 4.4 \times 10^{-2} \text{ m/s} \)): A: SOS, B: MORS. The dark lines indicate the location of the wall of the reactor and the draft tube (Varma and Al-Dahhan 2007).

Selected results of the azimuthally average mean liquid velocity values computed from the CARPT data are represented in the quiver plots in Figure 6-11. This figure depicts
the azimuthally averaged velocity vectors in the system formed due to its axial and radial components, hence depicting the flow pattern in the digester with the MORS and SOS systems (for $Q_g = 0.05 \times 10^{-3}$ m$^3$/s and $V_g = 4.4 \times 10^{-2}$ m/s in draft tube). As mentioned earlier, the system is divided into small cells for data processing; the vectors in this figure are shown for each such cell. It can be observed that two circulation loops exist. The liquid tends to rush into the bottom part of the draft tube in the region where the gas is introduced through the sparger, as is evident for both the MORS and the SOS system. It can be observed in this region that the velocity vectors are more prominent in the SOS system (Figure 6-11A). By virtue of the design of the SOS system, the gas is introduced at one single location into the system, which creates a local low density spot which sucks the liquid into this region. The liquid then is carried up the draft tube and gets released in the upper section of the reactor. The streamlines plot shown in Fig. 13 depict the direction of flow liquid in the system, these confirm the results shown in Figure 6-11 for the SOS and MORS system.

Similar observations were made by Karim et al. (Karim et al. 2004). Gas in the draft tube facilitates this process. With the momentum gained, the liquid is then pushed to the periphery of the reactor, as evident from the high radial component of the velocity vector in the upper part of the bioreactor (Figure 6-11). The liquid is then pulled back into the region around the exterior of the draft tube after it comes in contact with the wall of the reactor. Hence a recirculation eye is formed in the upper part of the reactor around the draft tube region, which suggests that the entire peripheral region of the reactor does not act as a downcomer. The velocity vectors are more uniform in the draft tube region for the MORS system, as they have a stronger axial component than in the SOS system. This suggests that the stream of gas bubbles tends to spiral around in the region between the SOS and the draft
tube; hence there is a significant radial component which prevents the arrows from being vertically oriented (Figure 6-11B). This phenomenon does not arise in the MORS system, as a better gas holdup distribution essentially reduces the radial components in the velocity vectors. Hence the flow is uniform in MORS as the arrows appear pointed upwards (Figure 6-11B). The flow pattern trends observed in the quiver plots are similar for both spargers to those observed in Figure 6-11 for other flow conditions (listed in Table 6-1), and hence they are not shown here.

**Axial liquid velocity**

![Figure 6-12](image)

Figure 6-12: Time averaged axial velocity comparison for A: level 1 (5x10^{-2} m) and B: Level 2 (150x10^{-2} m) from CARPT data. The gas flow rates have been indicated in parenthesis (Q1=0.017x10^{-3} m^3/s, Q2=0.050x10^{-3} m^3/s and Q3=0.083x10^{-3} m^3/s) (Varma and Al-Dahhan 2007).

Figure 6-12 shows the plot of the azimuthally averaged mean axial liquid velocity versus the radius of the system. These figures help visualize the values in quantitative terms.
It can be seen that the axial velocities in the system with MORS are higher than that with SOS system for any given gas flow rate at both level 1 and level 2. This is attributed to the higher mean gas holdup, as seen in Figure 6-8, and better gas distribution in the draft tube due to the lower values of $N_{Mal}$ (Figure 6-10) for the MORS system. The liquid velocities are higher at level 2 for any given sparger and gas flow rate. This is attributed to the fact that the gas distribution is better at level 2 for any given gas flow rate and sparger. The higher liquid velocity in the draft tube creates more circulation in the downcomer region, as can be seen in the increase in the negative axial velocity in the downcomer region of the reactor for higher gas flow rates. This increase is an indication of better circulation in the system in the system.

**Determining poorly mixed zones and root mean square (RMS) liquid velocity**

To better understand the effect of the sparger configuration on the mixing in the bioreactor a contour plot of root mean square (RMS) values of the radial, azimuthal and axial time averaged components of the velocity was prepared. The velocity contours help identify the zones in the system where the liquid velocity is such that it would cause the solids or bio flocs in the slurry to settle. Unfortunately, since the data for bio flocs settling characteristics of the slurry used in this study was not gathered during the experiments, the settling values available in literature were used to mark these contours. The terminal settling velocities for flocs from similar systems are reported in the range of $0.2 \times 10^{-3}$ m/s to $20 \times 10^{-3}$ m/s (Lee et al. 1996; Li and Yuan 2002. Based on Li and Yuan {Li, 2002 #12) Karim and Thoma (Karim et al. 2007) have used a settling velocity of $0.32 \times 10^{-2}$ m/s as a criteria for determining poorly mixed zones in gas-lift digesters. This same liquid velocity value has been used here to develop a slice of the velocity contours along the radial and axial direction of
the bioreactor as shown in Figure 6-13 for one of the gas flow rate condition studied. The regions of the reactor that has liquid velocity below the contour with a velocity $0.32 \times 10^{-2}$ m/s have been indicated in the darkest shade. This figure also depicts the streamline in the system generated from the velocity data obtained from the CARPT experiments using the streamlineslice function available with Matlab® software.

The streamlines clearly depict the circulation patterns observed in the bioreactor. The bottom part of the system with SOS shows zones with no streamlines indicating negligible liquid velocity. Some breaks appear in the streamlines in Figure 6-13. These could be due to the experimental error associated with the CAPRT technique and the fact that a two dimensional plane (r-z slice) for a given value of $\theta$ ($\theta=0^\circ$) is depicted. Hence, the broken streamlines are the ones that enter and leave this plane. The contour plots show greater fraction of the volume of the reactor under the velocity contour for $0.32 \times 10^{-2}$ m/s in the system with SOS. Table 6-2 summarizes the volumetric percentage of poorly mixed zones in the reactor for all the gas flow rates covered for both the system based on the three dimensional (r, $\theta$ and z) contour plots developed (not shown). These results clearly show that the poorly mixed zones double when a SOS system is used for a given gas flow rate. For the higher superficial gas velocity covered, the MORS reduces the poorly mixed zones by a factor of three. This is because the MORS system has lower $N_{Mal}$ value in the draft tube region for a given flow rate, this creates a higher density gradient between the draft tube region and the outside region within the reactor that increases the liquid circulation. The presence of poorly mixed zones would over a period of time encourage the settling of solids in the reactor which could lead to the drop in performance of the system. Since these types
of reactors have a high hydraulic retention time, over a period of time the system with SOS is likely to accumulate a lot of solids as compared to MORS.

Figure 6-13: Contour plots showing the time averaged values of the magnitude of the liquid circulation velocity for the bioreactor with 5% (TS) solid loading slurry at for gas flow rate $Q_g = 0.05 \times 10^{-3}$ m$^3$/s ($V_g = 4.4 \times 10^{-2}$ m/s): A: MORS, B: SOS. The dark lines indicate the location of the wall of the reactor and the draft tube; the fine lines with arrows indicate the stream lines. Numeric values in the box indicate velocity $\times 10^2$ (m/s) values the contour represents. The black colored zones indicate poorly mixed regions (Varma and Al-Dahhan 2007).

Table 6-2: Percentage of poorly mixed zone in bioreactor for the superficial gas velocities conditions studied for the bioreactor with MORS and SOS.

<table>
<thead>
<tr>
<th>Superficial gas velocity based on draft tube diameter $\times 10^{-3}$ (m/s)</th>
<th>Percentage dead volume in bioreactor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.47</td>
<td>MORS 11.52 %</td>
</tr>
<tr>
<td></td>
<td>SOS 24.91 %</td>
</tr>
</tbody>
</table>

1.47

4.41

6.46 %

13.27 %
The mean per cell, time averaged RMS velocity values obtained from all the cells in the draft tube region and the all the cells in the reactor (including draft tube region) for the reactor with MORS and SOS give an idea of the impact of gas flow rate on the liquid velocity. The mean time averaged RMS velocity for the draft tube region alone, as well as the entire reactor, is shown for the different superficial gas velocities in Figure 6-14. For a given gas flow rate, the MORS system gives higher RMS liquid velocities than the SOS for the entire reactor region and the draft tube. The difference is more pronounced in the draft tube region. This is again attributed to the lower values of $N_{Mal}$ obtained with the MORS system for a given gas flow rate.

Figure 6-14: Plot of superficial gas velocity is the draft tube versus time averaged average RMS liquid velocity in the draft tube for the MORS and SOS systems (Varma and Al-Dahhan 2007).
6.4 Conclusions

The viability of anaerobic bioreactors used for a source of energy generation from organic waste is strongly dependent on their energy economy. The net process energy consumed for proper operation should not exceed the energy available from the biogas (methane) generated. For optimal performance, mixing should be as efficient as possible. CARPT and CT were successfully implemented on a surrogate system to measure gas holdup distribution and liquid velocity and poorly mixed zones in a gas recirculated anaerobic bioreactor.

For a given power input, fixed by the gas flow rate (based on Equation 1), the Multi Orifice Ring Sparger (MORS) was found to give better gas phase distribution and higher mean gas holdup in the draft tube when compared to Single Orifice Sparger (SOS). All pores in the MORS do not open, conducting this study at gas flow rates that ensure all the MORS pores are open would have a power input range that would exceed the recommended range for anaerobic systems {EPA, 1979 #52. Lower values of $N_{Mal}$ confirm a better performance of the MORS in terms of gas holdup distribution characteristics.

The CARPT results determined that there are poorly mixed zones in the downcomer region of the reactor. Higher liquid velocity values were observed in the draft tube region for the bioreactor with MORS for a fixed gas flow rate. The poorly mixed zones are drastically reduced in the reactor when a MORS system is used as the lower values of gas holdup $N_{Mal}$ in the MORS ensures better liquid circulation. The same trend was observed with the RMS liquid velocity in the entire reactor. Hence the MORS system is considerably more efficient for mixing the reactor than the SOS system. The reduction in the poorly
mixed zones would make a larger impact when the bioreactors based on the configurations discussed here are scaled up for pilot plant operations.

Recirculation of the liquid is facilitated by the density gradient between the material in the draft tube and periphery area of the draft tube. This difference triggers the buoyancy forces that enable the liquid to circulate. High gas holdup and better distribution are therefore desirable in the draft tube region to create an effective density gradient for better mixing in the reactor by reducing the fraction of the poorly mixed zones. This objective could be easily achieved with an increase in the superficial gas velocity. However, the energy constraints in such systems give limited flexibility in manipulating the superficial gas velocity as an operation parameter. Hence this necessitates a careful consideration of the sparger design for introducing gas into the system. The impact of the increased and uniform gas holdup distribution in the draft tube, and the reduction in the poorly mixed zones in the bioreactor on the production of methane from bovine waste could be the subject of a performance study. This study confirms that for a given power input, efficiency in mixing can be obtained by appropriate sparger design.
Chapter 7

Performance Studies in Pilot Scale Anaerobic Bioreactor for Optimal Methane Generation

7.1 Introduction

Performance studies were carried out by Vesvikar {Vesvikar, 2005 #4} in a laboratory scale bioreactor where it was concluded that there was no significant effect of mixing in the laboratory scale. However, in the pilot scale bioreactor, operated at the same mixing power input, the effects of mixing were noted as significant. This was done by turning off the gas recirculation and noting the difference in methane generation between mixed and unmixed conditions. Vesvikar (Vesvikar et al. 2005) also carried out hydrodynamic studies using CARPT at various gas flow rates and computational fluid dynamics (CFD) studies on the pilot scale anaerobic bioreactor in cold flow conditions.

This chapter discusses the performance studies and gas holdup distribution studies have been carried out at the same gas flow rate conditions of the hydrodynamic studies carried out by Vesvikar (Vesvikar et al. 2005). The objective of the performance studies is to see the methane generation versus the mixing power input into the bioreactor and determine if there is an optimum. The performance studies have been carried out at the same Volatile (VS) solids loading, hydraulic retention time (HRT) conditions as those carried out by Vesvikar (Vesvikar et al. 2005).
7.2 Materials and Methods

Two pilot scale anaerobic bioreactors were constructed that were of the same dimensions. The first was constructed of steel and was designed for the performance studies. The second was made of acrylic with an open top for the hydrodynamic studies. The details of the reactor and the sparger used have been shown in Figure 7-1. The effect of sparger design studies on the laboratory scale bioreactor reported in Chapter 6 indicate that not all pores of the sparger open up. In the range of gas flow rates (power input) covered, sufficient pressure drop is unavailable for all opres to open. In this study the MORS sparger was redesigned to a cross sparger (CS) with four open pores as shown in Figure 7-1. The pores are located such that they lie on the circumference of a hypothetical circle that occupies half the area of the draft tube. This is to ensure gas distribution with in the draft tube. The draft tube to bioreactor diameter ratio was maintained the same as those reported in Chapter 6 (0.25). The draft tube is held in position with the bioreactor with the aid of three acrylic tie rods attached placed equiangularly with respect to the central axis of the bioreactor. These tie rods connect the draft tube to the wall of the bioreactor. In the lower section of the draft tube three legs are attach to the draft tube to prop it up as a tripod. The conical bottom has an exit port for removing the slurry from the bioreactor. The second port accommodates the gas line into the sparger. The sparger is placed such that pores are at the lower section of the draft tube. This way the gas is released in to the system at the bottom of the draft tube remains confined within the draft tube.

The upper lid covering the reactor was used for the performance studies alone and was not used for the hydrodynamic studies. The lid has ports for the biogas to escape, a feed port, and a port connected to a 10 lit gas bag that acts as a reservoir. Separate arrangements
for the gas line to the sparger were used in the bioreactor for hydrodynamics studies and the bioreactor for performance studies. The location of the cross sparger was the same. The gas line to the sparger in the bioreactor used was used for hydrodynamics study was fed from the bottom of the reactor as shown in Figure 7-1. The gas line for the performance studies bioreactor is introduced from the top and comes down to the sparger at the bottom of the draft tube. This arrangement is similar to the one show in Figure 6-1.

Figure 7-1: Schematic of the pilot plant anaerobic bioreactor used is shown. The arrangement of the gas line the sparger shown in this schematic was used in the bioreactor used for hydrodynamics studies.
7.2.1 Hydrodynamic studies

Tomography studies we carried out using the DE-DSCT equipment as discussed in Chapter 5. The bioreactor used in this study was operated in the surrogate mode similar to the bioreactors used for the sparger studies reported in Chapter 6. The cow manure as prepared for the performance studies was used in this study. An air line was connected to the gas line of the sparger. The air line has a rotameter to control the flow rate and a condenser attached. The operating conditions used were the same those described in Table 7-1.

The tomography scans were carried out at sections called out in Figure 7-1. Level 1 is just above the sparger in the lower part of the bioreactor, and Level 2 is just below the upper section of the draft tube. The contrast in attenuation available using the 661 keV (from $^{137}$Cs) and the 1332 keV (from $^{60}$Co) $\gamma$ photons at the VS loading of 6.6 % is very small. Since biosolids do not have elements with high atomic numbers, and the density of the solids is not a lot greater than the water, the attenuation is not very different from water at higher energies. Even at low energies of the $\gamma$ the contrast is expected to be better not significant to be discernable for this case. Simple densitometry measurements were made with the reactor filled with water and with the slurry. The transmission ratio of the counts was in the same range for both. Hence the peso – two phase assumption was applied.

Since the anaerobic bioreactor has a large diameter all the 15 detectors of the fan beams of in DE-DSCT setup are required such that they encompass the column. Hence a total of 315 projections per view and 197 source positions were used leading to a total of 62,055 (315 $\times$ 197) projections across the domain. The projections data was measured at a sample frequency of 5hz. The data was processed using the ME-AM algorithm as discussed in section 3.1.2 in Chapter 3. A $80 \times 80$ pixel resolution was used to the reconstruct the
images. The gas holdup distribution images were determined post processing using (3-29) as discussed in section 3.3 of Chapter 3.

### 7.2.2 Performance studies

![Schematic of pilot plant anaerobic digester and paraphernalia.](image)

Figure 7-2: Schematic of pilot plant anaerobic digester and paraphernalia.

A schematic of the setup used for the performance studies is shown in Figure 7-2. The pilot scale anaerobic bioreactor was housed in a temperature-controlled (35°C) cabinet. A line was connected to the port in the top plate covering the bioreactor. This port opens up in the head space of the bioreactor and allows for the biogas to escape. This line splits into two branches. One branch connects to a wet gas meter that continuously measures the volume of the gas as it vents out to the atmosphere. The other branch is connected to a gas compressor (GCA/Precision Scientific, Chicago, IL). This compressor collects the gas in the head space and pumps it to the sparger in the draft tube region of the reactor. The line connecting the compressor and the gas line to the draft tube has a condenser to trap the
moisture and a rota meter to monitor the gas flow rate. Another port on the top plate is connected to a gasbag. This gasbag acts as a reservoir or an extension of the head space in the bioreactor. The portion of the gas that is not recirculated in the bioreactor collects Tedlar gasbag. When the bag was full, the gas flow is directed through the gas meter that measures the volume before discharge.

The anaerobic bioreactor was operated using cow manure collected from Kraus dairy farm (Peevely MO) which used concrete bedding in the cow shed. The cow manure was obtained fresh (less than 7 days old) from cows under no antibiotic treatment. (Antibiotic treatment of cows limits the viability of methane generating microorganisms in the cow manure). The cow manure was then stored in a freezer and dispensed for the studies as per the requirements. Preparation of the manure involved a number of steps. The stored manure was thawed and blended with tap water (in 1:1 ratio, to adjust total volatile solids content) and placed into a large bucket for the heavy solids (sand, etc.) to settle out. The manure was then screened through a 1 cm pore seize sieve to separate large chunks of fiber and other forms of waste that has the potential to choke the digester. Then triplicate samples were taken to determine total solids (TS) by drying at 105°C for 24 hours, after which the samples were placed in an oven at 540°C for 60 minutes to determine total volatile solids (VS). Finally the slurry was diluted as needed with water to obtain 6.6% VS concentration (the manure has a TS of about 9-11% with very low sand content). Solids occur in the waste due to different sources like husk, straw, fiber and rice hulks etc coming from the feed and also from the kind of bedding used in cow sheds. Sand content was low as the shed has concrete bedding.

The digester was operated in fed batch matter (pseudo – continuous). Every alternate day, gas composition, and cumulative gas production volume were recorded. Feeding rate
(or effluent removal rate) was adjusted to maintain a hydraulic retention time (HRT) of 16 days. A known amount of reactor content (12 l) was removed and the same amount of feed was added through the feed port from the top.

The cumulative gas volume was determined by a wet gas test meter. Gas samples were collected through the sampling port using a gas-tight syringe to analyze the methane and carbon dioxide content using a GOWMAC gas chromatography system. TS were determined by drying a known weight of slurry at 105°C over night. TVS were determined by volatilization of a known weight of dried slurry at 540°C an hour.

Table 7-1: Details of the operating conditions used for the hydrodynamic and performance studies

<table>
<thead>
<tr>
<th>Gas flow rate $Q_g$ (l/min)</th>
<th>Superficial gas velocity in the draft tube (cm/s)</th>
<th>Energy input based on equation (6-1) (W/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4.5</td>
<td>0.731</td>
<td>4.25</td>
</tr>
<tr>
<td>9.0</td>
<td>1.462</td>
<td>8.49</td>
</tr>
<tr>
<td>18.0</td>
<td>2.923</td>
<td>16.99</td>
</tr>
</tbody>
</table>

The anaerobic bioreactor was run at the biogas recirculation rates as mentioned in Table 7-1. The anaerobic bioreactor was run from the highest down to the lowest gas glow rate. For the startup the process of the bioreactor the feed was mixed with the contents of the bioreactor left over from the studies of Vesvikar (Vesvikar et al. 2005). Initially natural gas was pumped into the reactor for recirculation. The Tedlar gasbag was filled with the natural gas and used as a buffer reservoir. It was replenished with the natural gas regularly and reattached to the reactor. Over a period of time (2 HRT periods) the biogas production
started and the replenishments of the natural gas were not required. The gas meter indicated the gas readings. The analysis of the gas sample indicated a rich methane and carbon dioxide content in the gas.

The study was started at the highest biogas recirculation rate listed in Table 7-1 to the lowest. For the first gas flow rate condition the bioreactor was run for a long duration (7-8 HRT periods) as the initial startup took some time to consistently generate biogas. Later when the system stabilized the gas recirculation rate was changed as per the objective of the study. For every gas flow rate the system would be run for 4-5 HRT periods. The last bio gas production data from the last 2 HRT periods for each gas recirculation rates was analyzed.

7.3 Results and Discussion

7.3.1 Hydrodynamic Studies

The gas holdup distribution images are shown in Figure 7-3. The images show that the gas holdup distribution is the draft tube region of the bioreactors at Level 1 and Level 2 for all the gas flow rates covered. These images are represented using a 21 × 21 pixel grid in the center of the 80 × 80 grid used to reconstructed gas holdup image for the entire domain. The results show that the gas is well distributed in the draft tube region of the reactor. The local gas holdup hot spots in the images at the lower level (Level 1) are the gas coming out of the sparger. The gas distributes well as it rises with in the draft tube. The average gas holdup values and the maldistribution number are for the gas flow rates studied are given in Table 7-2. The average gas holdup has been calculated using (6-3) and the maldistribution number has been calculated using (6-5).
Figure 7-3: Time averaged gas holdup distribution images in the draft tube region for Level 1 and Level 2 (called out in Figure 7-1). A: Gas holdup for $Q_g = 4.5$ l/min. B: Gas holdup for $Q_g = 9$ l/min. C: Gas holdup for $Q_g = 18$ l/min.

Table 7-2: The average gas holdup and maldistribution number of gas holdup in the draft tube region of the pilot scale bioreactor for the conditions studied (Table 7-1).

<table>
<thead>
<tr>
<th>Gas flow rate $Q_g$ (l/min)</th>
<th>Mean gas holdup in draft tube</th>
<th>$N_{mal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
<td>Level 2</td>
</tr>
<tr>
<td>4.5</td>
<td>0.017</td>
<td>0.029</td>
</tr>
<tr>
<td>9.0</td>
<td>0.033</td>
<td>0.0344</td>
</tr>
<tr>
<td>18.0</td>
<td>0.055</td>
<td>0.086</td>
</tr>
</tbody>
</table>
The results indicate that for a given gas flow rate the gas holdup is higher in the upper section of the draft tube (Level 2) than the lower (Level 1). The maldistribution number is lower in the upper section (Level 2) than in the lower section (Level 1) of the draft tube. This indicates that gas distributes well as it rises to the upper section of the draft tube. As the gas flow rate increases the maldistribution number reduces.

Table 7-3: Percentage of dead zones in the pilot scale anaerobic bioreactor

<table>
<thead>
<tr>
<th>Superficial gas velocity in the draft tube (cm/s)</th>
<th>Percentage dead volume in pilot scale bioreactor</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>13.63</td>
</tr>
<tr>
<td>9.0</td>
<td>8.19</td>
</tr>
<tr>
<td>18.0</td>
<td>5.20</td>
</tr>
</tbody>
</table>

The percentage of the poorly mixed zones or the dead zones based on the data above was determined from the liquid circulation velocity data from the CARPT studies carried out by Vesvikar (Vesvikar et al. 2005) for the conditions. The terminal settling velocities for flocs from similar biological systems are reported in the range of 0.2x10^-3 m/s to 20x10^-3 m/s (Lee et al. 1996; Li and Yuan 2002). Based on Li and Yuan (Li and Yuan 2002) Karim and Thoma (Karim et al. 2007) have used a settling velocity of 0.32x10^-2 m/s as a criteria for determining poorly mixed zones in gas-lift digesters. The same criteria was applied in Chapter 6. This criterion was applied to the CARPT based velocity data of Vesvikar (Vesvikar et al. 2005). The results are shown in Table 7-3. The percentages of dead zones are in the range of what was obtained for laboratory scale bioreactor with MORS.
There are far less than the percentage obtained for the SOS system. This indicates that system has successfully scaled up to the pilot scale in terms of the mixing.

### 7.3.2 Performance Studies

![Bidaily methane production for the different biogas recirculation rates covered over the period of the study. The gas volume is corrected to standard temperature and pressure.](image)

Figure 7-4: Bidaily methane production for the different biogas recirculation rates covered over the period of the study. The gas volume is corrected to standard temperature and pressure.

The biogas production was monitored on a bidaily basis. The gas sample was analyzed for methane and carbon dioxide composition using gas chromatography (GC) on a bidaily basis. The composition of the biogas was determined against a calibration curve for a methane and carbon dioxide mixture. Based on the GC readings the bidaily methane content in the biogas was determined. This value has been plotted for the entire duration of the study in Figure 7-4. The zone of data points for different gas recirculation rates have been
indicated in Figure 7-4. The anaerobic bioreactor was at the first gas recirculation condition for the maximum duration. It can be seen that the gas production took some time to rise and stabilized. After as the gas recirculation rate was reduced a drop in the methane production is noticed. There was methane generated at the unmixed condition where there was no gas recirculated.

![Graph showing cumulative methane production](image)

Figure 7-5: Cumulative methane production of the last two HRT periods when the system is in steady state for the gas recirculation rates covered. The methane volume is corrected to standard temperature and pressure.

From the data it is clear that there is discrepancy in the bidaly volume of methane produced. This is typical of such open systems. Although the VS content of the feed was kept consistent, there are natural variations in the feed that affect the outcome. In order to analyze it better, the methane production data for the last two HRT periods of each gas recirculation was used to plot the cumulative methane production. For every gas flow rate covered, the system is run for 4-5 HRT periods as the system take time to reach a steady state. The system is most likely to be in steady state in the last two HRT periods. The
cumulative plots are shown in Figure 7-5. Clearly the different slopes of the cumulative production curved indicate the there is significant effect of mixing on methane generation in bioreactor for the range of gas flow rates covered.

Figure 7-6: A plot of average daily methane production versus the power input for mixing. The gas volume is corrected to standard temperature and pressure.

To get a perspective on the rate of steady state methane generation the mean value of the methane generated is on a daily basis is plotted versus the power input for mixing as shown in Figure 7-6. The power input is calculated from the gas flow rates and the values are given in Table 7-1. The mean and standard deviation are calculated from the same data used to generate the cumulative methane generation plots. Figure 7-6. shows that the relationship between mixing and methane production is not linear. The trend suggests that at higher mixing rates the methane generation levels off and reaches an asymptote. The interesting to note that that at zero gas recirculation rates the system is still producing a significant amount of. The only mixing it receives is from the effluent removed from the
bottom of the reactor and the feed added from the top on a bidaily basis. The results seems to indicate there is an optimum rate for mixing such that for a minimal gas recirculation rate the maximum methane could be generated.

7.4 Remarks

The results of the tomography study shows that the gas is well distributed within the draft tube region of the system. The liquid velocity increases and the dead zone are minimized (Table 7-3). The performance studies indicate the reduction in the dead zones within the anaerobic bioreactor helps improve the performance of the anaerobic bioreactor to increase the gas generation. At the high gas flow rate there are still dead zones present within the system. Visual observation indicates certain degree of settling of solids at the bottom of the reactor. Increasing the gas flow rates further will certainly help improve the liquid recirculation with the bioreactors, and prevent solids sand and other form of debris from settling. However, the gain in terms of methane generation will be marginal at best.

This suggests that better pretreatment strategy of the feed to eliminate solid debris may be useful. The solid debris may not be avoidable in all the cases especially if the manure is from a cow shed that uses sand bedding. In such situation it would be advisable to modify the operation of the digester such that solids that settled at the bottom are drained out. The inert content could be separated and disposed and the organic matter could be recycled in the feed. Clearly from a performance or methane generation point of view power input rates higher than the 16-20 W/m³ will not provide any benefit. From a commercial perspective higher energy input to generate more methane is not very helpful. The higher power input could help manage the system better in terms of prevention of choking etc if a pretreatment process or solids elution strategy is not used for field operation of the anaerobic bioreactor.
Chapter 8

Summary and Recommendations

8.1 Summary

Treatment of animal waste in anaerobic bioreactors reduces environmental pollution while generating methane for use as fuel. Proper understanding of anaerobic bioreactor's performance and the effects of the variables that influence its performance is necessary. Mixing is one of the important variables affecting digester performance. Bio-gas mixed digesters were studied as they offer many operational advantages over other mixing configurations. The phase holdup distribution in the flow affects the performance of anaerobic bioreactors. Tomography was used to study the effect of operating parameters and design conditions on phase holdup distribution noninvasively. And the effect of the phase holdup distribution of the liquid circulation velocity and mixing studied using the computer automated particle tracking technique. The main objective of this work was to develop and validate a dual energy – dual source computed tomography (DE-DSCT) technique such that it is capable of imaging phase holdup distribution in a wide range of three phase systems, and utilize it to further the understand the hydrodynamics of anaerobic bioreactor and its effect on the performance.

The important findings related to various aspects of this dissertation have been summarized in this chapter. These finding have been discussed in detail in the last sections of Chapter 3 through Chapter 7.
8.1.1 Algorithms for tomography of multiphase systems

The image processing algorithm is one of the key aspects of any tomography technique. In this study algorithms related to the γ ray CT for two phase and three phase flow were studied.

Two phase flow:

The alternating minimization (AM) algorithm was implemented for single source γ ray CT. A detailed comparative study of the expectation maximization (EM) and AM algorithms was done for the single source γ ray CT with simulated and experimental transmission data. The holdup images were generated using the post processing from the attenuation images reconstructed with the algorithms. The key findings are as follow:

- The phase holdup images from AM algorithm were more accurate than the EM algorithm images. The difference was more pronounced in the gas – liquid domains with high attenuating internals than the domain with the gas – liquid domain.
- The holdup images generated with EM algorithm were more grainy than the images from the AM indicating that the AM algorithm can handle noise in the data better. As the number of iterations increase the noise levels in the AM images are more stable then the EM images.
- The difference in performance, particularly when high attenuating material are present in the domain, is attributed to the simplifications made in deriving the update function of the EM algorithm. The AM algorithm doesn’t use any such simplification.
- Both the stochastic algorithms (AM and EM) out perform the deterministic filter back projection algorithm commonly used in imaging applications.
Three phase flow:

For imaging three phase flow transmission data from two different γ energies is required to have a closed set of equations. With the objective of imaging large scale system in mind two high energy γ photons were selected as the basis. Projections data was simulated for a synthetic three phase phantom using the attenuation coefficient data of the material in the phantom at the energies of the γ photons selected. Phase holdup images were generated following the post processing monoenergetic approach. The holdup images generate in this manner had an unacceptable amount of error. The polyenergetic AM (PE-AM) was applied to determine the holdup images directly avoiding a post processing approach. The key findings are listed below:

- The monoenergetic approach to the dual energy CT problem, which is a liner extension of the approach followed for two phase systems, provides high erroneous images. A lot of the pixels in these images have holdups either negative or greater than unity.

- The high error is attributed to ill conditioned system of closed liner equations used to determine the holdup images from the attenuation images. The ill conditionally is attributed the matrix that has the attenuation coefficient of the phases in the flow.

- The high accuracy of the attenuation image of the three phase flow generated with the monoenergetic AM algorithm is insufficient to produce accurate phase holdup images. Only possibility to improve the image quality is use a very low energy γ photon where the attenuation coefficient matrix of the unmixed phases is not so poorly conditioned. This would mean only very small domains can be scanned.
The polyenergetic AM (PE-AM) algorithm is able to generate high quality holdup images where the ME-AM fails. The PE-AM is not affected by the highly ill conditioned cases unlike the ME-AM. This is because the PE-AM doesn’t rely on the post processing approach and determine the phase holdup; it computes it directly thereby allowing for corrections within the iterative process.

The PE-AM algorithm is slow in convergence, has a higher computational time, and requires much iteration. The convergence is dependant on the attenuation coefficients values of the phases in the mixed flow. If the contrast available is low, it takes longer to converge.

With the PE-AM algorithm the phase holdup image of the higher attenuating material in the flow convergence faster than the other images. This is attributed uneven contour of the I-divergence. The I-divergence is the cost function that is minimized to reconstruct the holdup images.

8.1.2 Development of the DE-DSCT technique and its validation

The hardware for the DE-DSCT scanner was fabricated and assembled in CREL. This setup was automated with stepper motors. The SCDs that house the sources were developed by the Dr. Charles Alexander at ORNL; the data acquisition hardware was developed by Dr. Allen Winterberg a ORNL. This system is designed to image column up to 30 inches in diameter and 9 feet in height. This system was validated with a three phase phantom. The PE-AM algorithm was used to determine the phase holdup distribution images. The key conclusions are listed below:

- The data acquisition system used is advanced and is able to avoid Compton scatter in the measurements. It could be used for radioisotopes with up to 8 photo peaks.
• The DE-DSCT was validated with a three phase phantom. For the first time experimental evidence was provided to prove that the high energy $\gamma$ photons can be used to image three phase system. Since high energy $\gamma$ photons have a high penetration depth, the DE-DSCT can image large scale three phase flow systems.

• The PE-AM algorithm works with actual experimental transmission from high energy $\gamma$ data and is able to provide high quality holdup images with real experimental data.

• The DE-DSCT has a poor temporal resolution but has high spatial resolution (up to 2 mm).

• As an obvious extension of this demonstration: if photo peaks with energies lower than the $^{137}$Cs isotope are chosen the accuracy in the results is only expected to improve further. Hence, the DE-DSCT and the PE-AM algorithm provides a basis for $\gamma$ dual energy CT where photo peaks from different isotopes could be used.

8.1.3 Effect of gas holdup and hydrodynamics of gas-lift anaerobic bioreactor.

Based on a critical review of various mixing configuration reported by Vesvikar (Vesvikar et al. 2005) the gas-lift anaerobic digesters were found to be effective in terms energy requirements and ease of operations over other configuration. Design parameter of the gas-lift anaerobic bioreactors such as effect of draft tube diameter and shape of the bottom section were studied using CAPRT and CFD simulations by Vesvikar (Vesvikar et al. 2005). In this study two different spargers design concepts were considered for a laboratory scale gas-lift anaerobic digester: the single orifice sparger (SOS) and the multi-orifice ring sparger (MORS). The effect of sparger design on the gas holdup in the draft tube was
studied using CT. The effect of the gas holdup distribution with in the draft tube on the liquid circulation velocity within the anaerobic bioreactor was studied using CARPT. The key findings were summarized as follows:

- For a same gas flow rate the CT results indicate the bioreactor with MORS provides a high gas holdup in the draft tube region than the bioreactor with SOS.

- For a given superficial gas velocity, the maldistribution number $N_{\text{mal}}$ has lesser values for the MORS than the SOS. This indicates that the gas distribution is more uniform in the draft tube region of the bioreactor with MORS than the bioreactor with SOS.

- The holdup results indicate that some of the lower gas flow rates not all the pore of the MORS open up as sufficient pressure drop doesn’t exist. In spite of this the MORS has a better gas distribution than the SOS.

- The CARPT studies indicate that the higher gas distribution with the draft tube leads to a higher liquid velocity with in the draft tube region and the down comer region of the bioreactor. The over all liquid circulation is improved with better gas distribution.

- For a given superficial gas velocity, and by extension power input for mixing, the dead zones or zones that are poorly mixed are reduced drastically in the bioreactor with the MORS as compared to the bioreactor with SOS.

- For a given mixing power or energy input, the system with the MORS is more efficient in mixing the bioreactor than the system with the SOS.
8.1.4 Optimization of pilot scale anaerobic bioreactor for methane generation

Based on the hydrodynamic studies done for the laboratory scale anaerobic bioreactor a pilot scale bioreactor was designed with an active volume of 100 l and with geometric similarity. The MORS was redesigned to reduce the number of pores as not all the pores open up when it was used in the lab scale studies. The pilot scale rector was made to have a cross sparger with four open pores. Two versions of this reactor were made, one for performance studies and the other for the hydrodynamic studies. Preliminary performance studies were done at ORNL to check the effect of mixing and scale up in terms of performance. These have been reported by Vesvikar (Vesvikar et al. 2005). The second reactor was used in the surrogate mode for detailed CARPT studies by Vesvikar (Vesvikar et al. 2005) on the effect of draft tube in the scaled up reactor.

In this study performance studies we carried out at different biogas flow rates over a in the bioreactor period of time to see the effect of mixing on methane generation at large scale. The superficial gas velocity and mixing power input were maintained the same as the laboratory scale bioreactor. The second bioreactor was used for the hydrodynamic studies. The gas holdup distribution within the draft tube region of the reactor was determined using the DE-DSCT setup. Some of the key findings are discussed here:

- Based on the steady state gas generation data, the increase in gas recirculation velocity increase average methane production. There was some methane generation at the unmixed condition (zero gas flow rate).
• The methane generation trend seems to level off at higher gas flow rates suggesting that an optimum point the maximum methane can be generated at a minimum power input or gas recirculation rate.

• Higher gas flow increase the mixing intensity, clears out the dead zones, and ensure that all the solids with in the animal waste slurry floats. However, it fails to provide any benefit in terms of biogas generation.

• There is indication that the solids related to waste would settle within the dead zone accumulate. A better strategy seems to periodically elute the settled solids for recycle or disposal without out disturbing the operation of the anaerobic digester as these solids may not be contributing to the gas generation.

8.2 Recommendations for Future Work

A key contribution of the dissertation has been the successful development of the DE – DSCT technique and it validation by actual experimental data. This has the potential to open doors to research in the area of hydrodynamics of a number of multiphase phase systems that are large scale and have three phases in the flow. The data generated could help improve the fundamental understand of the various design and operation parameters that effect the three phase flow. Vesvikar (Vesvikar et al. 2005) has mentioned a number of area of future research pertaining to the hydrodynamics of the anaerobic bioreactors. These have not been repeated here. The recommendations here have been restricted to those related to tomography. There are two set of future recommendations suggested here: improvement of the basic imaging technique and other in the area of application of the DE-DSCT.
8.2.1 Improvements in the tomography technique

Use of $^{75}$Se radio isotope for three phase imaging:

The $^{75}$Se radioisotope is polyenergetic $\gamma$ source that has five discrete photo peaks with a low and high energy range. It could be directly plugged into the DE-DSCT system developed. I would require only one fan beam as all the energies of $\gamma$ photons required for imaging is available from one single source. The PE-AM algorithm could be applied directly to process the holdup images. The data from this source could have some Compton scatter challenges which could be minimized in the DE-DSCT system as the data acquisition hardware allows for setting of energy windows. Five such windows could be set to get the counts data from 5 peaks separately. The major drawback of the source is that it has a smaller half life as compared to $^{137}$Cs and $^{60}$Co.

Work was initialed to apply for a license for this source with the radiation safety at Washington University. AEA technology manufactures the $^{75}$Se source and it is available in high strengths. A special SCD for this source was acquired from Tracer Co. This SCD is currently lying unused with CREL. In future, if a license is granted, the SCD could be shipped to AEA technology to insert a $^{75}$Se source. The lower energy photons from the $^{75}$Se source could image phase holdup distribution in systems where there is very little attenuation contrast available between the phases in the flow. Of course as a trade off the domain size may have to me smaller than those that can be scanned in the DE-DSCT, however they certainly can be larger than those demonstrated by Froystein et al (Froystein et al. 2005).
Real time imaging with good spatial resolution:

The DE-DSCT scanner discussed in this dissertation has a high spatial resolution and has a poor temporal resolution. There is always a trade-off between temporal and spatial resolution in tomography scanners. The main limitation in achieving this goal is the scanner hardware and not the algorithms or the computation infrastructure image reconstruction.

For real-time image of any flow the tomography scanner must be able to obtain a sufficient number of line measurements or projections of the data at a time scale that is far smaller than the timescale at which an event occurs in the flow. For most multiphase systems this would mean a time scale in the millisecond or microseconds. The number of projections measurements must be sufficient to provide spatial resolution such that the images, if not accurate, at least are acceptable enough to provide qualitative measurements. For most of the real time scanners reported in literature, the number of projections measurements are far less than the number of pixels used to reconstruct the images. This leads to an underdetermined system of equations.

The key to getting many line measurements is to have multiple fan beams in fixed positions each with a dedicated source (Johansen 2005; Mudde et al. 2005) or have the domain rotate at a high speed (Hampel et al. 2007a). The number of projection measurement these systems can measure in turn can be limited by the dimensions of the detector crystals. The crystals need to be large to absorb the photons of high energy. Higher energy detectors that are efficient enough to be small in size such many could be ‘packed’ into a one fan beam would help increase number of projection measurements. Hampel (Hampel et al. 2007a) has used small high energy detectors in a single source moving scanner. With the advancement in detector technology a new fixed scanner could designed such that I could provide many
more line measurements than the ones reported in literature to provide high resolution real
time images.

3D imaging:

The DE-DSCT scanner is a 2D scanner such that it images the phase holdup
distribution at a fixed vertical position. The base plate could be moved at multiple locations
to get more sectional images. If a cone beam used with a rectangular panel of high energy
detectors that are small then it is possible to make line or projection measurements across
the entire column. This data could be used to reconstruct a voxel based image of the entire
system. The cone beam and detector could be mounted on a circular rotating plate to get
multiple views. If multiples cone beam sources are used then a fixed scanner could be
developed that would provide 3D real time imagining.

8.2.2 Application of DE-DSCT to multiphase flow systems

There has been extensive research in area gas – liquid – solid flow in CREL
pertaining to slurry bubble column (SBC) (Rados 2003a; Shaikh 2007)and in stir tanks
reactor (STR). The DE-DSCT can be applied to study effect of operating parameters, design
parameter and flow conditions on the phase holdup distribution. The validity of constant
solid holdup in the liquid – solid part of the gas – liquid – solid flow made by Rados et al
(Rados et al. 2005a) Razzak et al(Razzak et al. 2007)and Warsito et al.(Warsito and Fan
2003)can be ascertained for different operating conditions of the SBC.

Some preliminary work has been done in this direction. DE-DSCT measurements
for different sold loading conditions and superficial has velocities have been made for a 4 in
diameter SBC. This data will be processed to determine the phase holdup distribution of the
three phases and a paper will shortly be published.
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